Stat873 HW01

1. 1 on p223

   In linear model $y = X\beta + e$, $X = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Show that $c_1\beta_1 + c_2\beta_2 + c_3\beta_3$ is estimable if and only if $c_1 = c_2 + c_3$.

   Hint: With $c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$, show that $c_1 = c_2 + c_3 \Longleftrightarrow c' = LX$ for some L.

2. 2. p223

   In $y = X\beta + e$, $\beta \in R^p$. Let $H = \begin{pmatrix} h'_{(i)} \\ \vdots \\ h'_{(p-1)} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{pmatrix} \in R^{(p-1)\times p}$, $1_p \in R^p$ and $c = \begin{pmatrix} c_1 \\ \vdots \\ c_p \end{pmatrix} \in R^p$. Show that

   $$H\beta \text{ is estimable} \Longleftrightarrow \text{“}1_p'c = 0 \Longrightarrow c'\beta \text{ is estimable''}$$

   Hint: $\mathcal{N}(1_p')$ has dimension $p-1$.
   But $h_{(i)} \in \mathcal{N}(1_p')$ and $h_{(1)}, ..., h_{(p-1)}$ are linearly independent.
   So $[h_{(1)}, ..., h_{(p-1)}]$ is a basis of $\mathcal{N}(1_p')$. Hence $\mathcal{N}(1_p') = \mathcal{R}(H')$.