

For $X \sim \text{Bernoulli}(p)$ based on a sample of size $n = 20$ one designed a test

$H_0 : p = 0.5$ versus $H_a : p \neq 0.5$
 Test Statistic $T = X_1 + \cdots + X_{20}$
 Reject H_0 if $T \leq 5$ or $T \geq 15$

1. Express $\beta(p)$ through Binomial distribution $B(n, p)$.

$T = X_1 + \cdots + X_{20} \sim B(20, p)$.
 So $\beta(p) = P(B(20, p) \leq 5) + P(B(20, p) \geq 15)$.

2. Find the significance level of the test.

Use link on www.math.wichita.edu/~xhu to Binomial distribution calculator.

$$\beta_T(0.5) = P(B(20, 0.5) \leq 5) + P(B(20, 0.5) \geq 15) = 0.02069 + 0.0207 = 0.04139.$$

The significance level of the test is $\alpha = 0.04139$.

3. Find the power of the test at $p = 0.8$.

$$\beta_T(0.8) = P(B(20, 0.8) \leq 5) + P(B(20, 0.8) \geq 15) = 0 + 0.80421 = 0.80421.$$

The power of the test at $p = 0.8$ is 0.80421.

4. Find the probability of Type II error at $p = 0.3$.

$$\beta_T(0.3) = P(B(20, 0.3) \leq 5) + P(B(20, 0.3) \geq 15) = 0.41634 + 0.00005 = 0.41642.$$

The probability of Type II error at $p = 0.3$ is $1 - \beta_T(0.3) = 1 - 0.41642 = 0.58358$.