Stat871 **HW10**

1. $X_1, ..., X_n$ is a random sample from $N(\mu, \sigma^2)$. Re-parameterize $\theta = \sigma^2$ and $\tau = \frac{\mu}{\sigma^2}$.

   (1) Write $f(x_1, ..., x_n; \theta, \tau)$ as $\exp\left[p(\theta, \tau) + q(x_1, ..., x_n) + r(\theta)T(x_1, .., x_n) + \tau S(x_1, ..., x_n)\right]$. Identify $p(\theta, \tau)$, $q(x_1, ..., x_n)$, $r(\theta)$, $T(x_1, .., x_n)$ and $S(x_1, ..., x_n)$.

   (2) Identify a sufficient statistic for θ and a sufficient statistic for τ.

   (3) Show that with respect to θ, the likelihood function has monotone ratio in $T(X_1, .., X_n)$.

2. For conditional α-level UMP test on $H_0 : \theta \leq \theta_0$ versus $H_a : \theta > \theta_0$ with θ in 1, the conditional pdf of T given S, $f_{T|S}(\cdot) = \frac{f_{(T,S)}(t, s; \theta, \tau)}{\int_t f_{(T,S)}(t, s; \theta, \tau)\, dt}$ is needed.

   Express $f_{(T,S)}(t, s; \theta, \tau)$ via $f_{N(\mu, \sigma^2/n)}(\cdot)$, the pdf of $N\left(\mu, \frac{\sigma^2}{n}\right)$ and $f_{\chi^2(n-1)}(\cdot)$, the pdf of $\chi^2(n-1)$.

   Hint: $\overline{X} = \frac{\sum X_i}{n} \sim N(\mu, \sigma^2/n)$ and $\frac{\sum_i X_i^2 - n\overline{X}^2}{\sigma^2} \sim \chi^2(n-1)$ are independent.