Stat871 **HW09**

1. For $H_0 : \theta \leq 0$ versus $H_a : \theta > 0$
   where θ is in the pdf of a logistic distribution $f(x;\theta) = \frac{e^{-(x-\theta)}}{[1+e^{-(x-\theta)}]^2}$, $-\infty < x < \infty$.
   based on a sample $X_1, ..., X_n$ from X with this logistic distribution,

$$\phi(U) = \begin{cases} 1 & U > c \\ 0 & U \leq c \end{cases} \quad \text{with } E_0[\phi(U)] = \alpha$$

   is a locally most powerful α-level test at $\theta = 0$. Find U.

2. $g(x) = f(x) - k_1 f_1(x) - k_2 f_2(x)$ where $k_1 > 0$ and $k_2 > 0$. Let

$$\phi(x) = \begin{cases} 1 & g(x) > 0 \\ r & g(x) = 0 \\ 0 & g(x) < 0 \end{cases} \quad \text{with} \int \phi(x) f_1(x) dx = c_1 \text{ and } \int \phi(x) f_2(x) dx = c_2.$$

   Suppose $0 \leq \psi(x) \leq 1$, $\int \psi(x) f_1(x) dx \leq c_1$ and $\int \psi(x) f_2(x) dx \leq c_2$. Show that if $\int \psi(x) f(x) dx = \int \phi(x) f(x) dx$, then $\int \psi(x) f_i(x) dx = \int \phi(x) f_i(x) dx$ for all $i = 1, 2$.
   Hint: Let $h(x) = \phi(x) - \psi(x)$ and examine
   $\int h(x) f(x) dx = \int h(x) g(x) dx + k_1 \int h(x) f_1(x) dx + k_2 \int h(x) f_2(x) dx$