Stat871 **HW07**

1. $X1, ..., X_n$ is a random sample from $N(\mu, 1^2)$.

   (1) Find statistic $T(X)$ such that the pdf of sample X is $f(X;\, \mu) = \exp\left[p(\mu) + q(X) + \mu T(X)\right]$.

   (2) Find the distribution of $T(X)$.

2. Sample X has pdf/pmf $f(X;\, \theta) = \exp\left[p(\theta) + q(X) + \theta T(X)\right]$ such that the sufficient statistic $T(X)$ has pdf/pmf $f(t;\, \theta) = a(\theta)b(t)e^{\theta t}$.

   Let $g_1(t) = f(t;\, \theta_0)$ and $g_2(t) = tf(t;\, \theta_0)$. Then the system of two equations (the two conditions for UMP for two-sided alternative hypothesis) $\begin{cases} \int_t \phi(t) f(t;\, \theta_0)\, dt &= \alpha \\ \int_t \phi(t)[f(t;\, \theta_0)]'_\theta\, dt &= 0 \end{cases}$ is equivalent to $\begin{cases} \int_t \phi(t) g_1(t)\, dt = \alpha \\ \int_t \phi(t) g_2(t)\, dt = c \end{cases}$. Find c.