

1. Suppose $E_{\theta}(\hat{\theta}_n) = \theta$ and $\text{Cov}_{\theta}(\sqrt{n} \hat{\theta}_n) \rightarrow I^{-1}(\theta)$.
Show that $\hat{\theta}_n$ is an asymptotically efficient estimator for θ .
2. $X_n = \begin{cases} 0 & p = 1 - \frac{1}{n} \\ n & p = \frac{1}{n} \end{cases}$ and $X \equiv 0$. Then X has cdf $F(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$ and $E(X) = 0$.
 - (1) Find cdf $F_n(x)$ for X_n .
 - (2) Show that $X_n \xrightarrow{d} X$.
 - (3) Show that $E(X_n) \not\rightarrow E(X)$.