Stat871 HW02

1. $\begin{pmatrix} y \\ y_* \end{pmatrix} \sim \left(\begin{pmatrix} X \\ X_* \end{pmatrix} \mu, \begin{pmatrix} \Sigma & C \\ C' & V \end{pmatrix} \right).$

   (1) Find expression for $\mathrm{Cov}(By - y_*)$.

   (2) Suppose $T = B + H(I - XX^+)$, $D = (B\Sigma - C')(I - XX^+)$. Show that

   $$\mathrm{Cov}(Ty - y_*) - \mathrm{Cov}(By - y_*) = HD' + DH' + H(I - XX^+)\Sigma(I - XX^+)H'.$$

   (3) Argue that if $(B\Sigma - C')(I - XX^+) = 0$, then $\mathrm{Cov}(Ty - y_*) - \mathrm{Cov}(By - y_*) \geq 0$.

2. $\overline{X}_n$ is the mean of a sample of size n, and μ is the population mean.

   (1) Show that $\frac{n}{n+k}\overline{X}_n$ is an asymptotically unbiased estimator for μ.

   (2) Show that $\frac{n}{n+k}\overline{X}_n$ is a consistent estimator for μ.