

1. File Table94.txt contains variables  $y, x_1, x_2, x_3, x_4, x_5, x_6$ . Consider model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon, \epsilon \sim N(0, \sigma^2)$$

and null hypothesis  $H_0 : \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \beta = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ .

- (1) Find SSE, DF of SSE, MSE,  $SSII_0$  and  $SSII_3$ . (Use SAS)

$$\begin{aligned} \text{SSE} &= 97.65789, & (\text{DF of SSE}) &= 8, & \text{MSE} &= 12.20724 \\ \text{SSII}_0 &= 212.25159, & & & \text{SSII}_3 &= 10.94040. \end{aligned}$$

- (2) Find the model reduced by  $H_0$ .

$$H_0 : \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \beta = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff \begin{cases} \beta_0 + 2\beta_1 - \beta_2 = 0 \\ \beta_2 - \beta_3 = 0 \end{cases} \iff \begin{cases} \beta_0 = \beta_2 - 2\beta_1 \\ \beta_3 = \beta_2 \end{cases}$$

Under  $H_0$ ,

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon \\ &= \beta_2 - 2\beta_1 + \beta_1 x_1 + \beta_2 x_2 + \beta_2 x_3 + \epsilon = \beta_1(x_1 - 2) + \beta_2(x_2 + x_3 + 1) + \epsilon. \end{aligned}$$

So the reduced model is

$$y = \beta_1(x_1 - 2) + \beta_2(x_2 + x_3 + 1) + \epsilon, \epsilon \sim N(0, \sigma^2).$$

- (3) For the reduced model find  $\text{SSE}_r$ , (DF of  $\text{SSE}_r$ ) (Use SAS).

$$\text{SSE}_r = 353.11014 \quad \text{and} \quad (\text{DF of SSE}_r) = 10.$$

2. Consider the model in 1.

- (1) Let  $F_0$  and  $F_3$  be the test statistics for testing on  $H_0 : \beta_0 = 0$  and  $H_0 : \beta_3 = 0$  respectively. Based on the results in (1) of 1, calculate the values of  $F_0$  and  $F_3$ .

$$F_0 = \frac{\text{SSII}_0}{\text{MSE}} = \frac{212.25159}{12.20724} = 17.38735 \quad F_3 = \frac{\text{SSII}_3}{\text{MSE}} = \frac{10.94040}{12.20724} = 0.89622.$$

- (2) Let  $F$  be the test statistic for testing on  $H_0$  in 1. Based on the results in (1) and (3) of 1, calculate the value of  $F$ .

$$F = \frac{(\text{SSE}_r - \text{SSE})/q}{\text{MSE}} = \frac{(353.11014 - 97.65789)/(10 - 8)}{12.20724} = \frac{127.72613}{12.20724} = 10.46$$

(3) Find the table by SAS for testing  $H_0$  in 1 to verify your calculation in (2) of 2.

	DF	MS	F	$Pr > F$
Numerator	2	127.72613	10.46	0.0059
Denominator	8	12.20724		