Stat 763 HW05

1. R^2 and R^2_{adj} are the coefficient of determination and the adjusted coefficient of determination for model $y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \epsilon$ based on a sample of size n.

   (1) Find a formula expressing R^2_{adj} via R^2.

   $1 - R^2 = \frac{SSE}{SSTO}$ and $1 - R^2_{adj} = \frac{MSE}{MSTO}$.

   So $1 - R^2_{adj} = \frac{MSE}{MSTO} = \frac{SSE}{SSTO}\frac{n-1}{n-(k+1)} = (1 - R^2)\frac{n-1}{n-(k+1)}$.

   Thus $R^2_{adj} = 1 - \frac{n-1}{n-(k+1)}(1 - R^2)$.

   (2) Find a formula expressing R^2 via R^2_{adj}

   $1 - R^2 = \frac{SSE}{SSTO}$ and $1 - R^2_{adj} = \frac{MSE}{MSTO}$.

   So $1 - R^2 = \frac{SSE}{SSTO} = \frac{MSE}{MSTO}\frac{n-(k+1)}{n-1} = (1 - R^2_{adj})\frac{n-(k+1)}{n-1}$.

   Thus $R^2 = 1 - \frac{n-(k+1)}{n-1}(1 - R^2_{adj})$.

2. 3.5 p126
   Consider model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_6 + \epsilon$ with data in Table B.3 on p576 also in file B3.txt with observations on y, x_1 and x_6 only.

   (1) (d) Find a 95% C. I. for β_1.

   $$\begin{aligned}\widehat{\beta}_1 \pm t_{0.025}(29) S_{\widehat{\beta}_1} &= -0.05302 \pm 2.045 \times 0.00615 = -0.05302 \pm 0.01258 \\ &= (-0.0656,\ -0.04044)\end{aligned}$$

   is a 95% C. I. for β_1.

   (2) (f) Find a 95% C. I. on the mean gasoline mileage when $x_{01} = 275$ in^3 and $x_{06} = 2$ barrels.

   $$\begin{aligned}\widehat{y}(x_0) \pm t_{0.025}(29) S_{\widehat{y}(x_0)} &= 20.1872 \pm 2.045 \times 0.6448 = 20.1872 \pm 1.3186 \\ &= (18.8684,\ 21.5061)\end{aligned}$$

   is a 95% C. I. for $E(y)$ when $x_1 = 275$ and $x_6 = 2$.

   (3) Find a 90% upper-sided confidence interval for mean gasoline mileage when $x_{01} = 275$ in^3 and $x_{06} = 2$ barrels.

$$\begin{aligned}(\widehat{y}(x_0) - t_{0.1}(29)S_{\widehat{y}(x_0)},\, \infty) &= (21.1417 - 1.311 \times 0.6448,\, \infty) \\ &= (21.1417 - 0.8453,\, \infty) = (19.3416,\, \infty)\end{aligned}$$

is a 90% upper-sided C. I. for mean gasoline mileage when $x_{01} = 257$ and $x_{06} = 2$.