

1. The values of variables y , x_1 and x_2 in Table3.2 on p76 are stored in file Table32.txt. Consider model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$, $\epsilon \sim N(0, \sigma^2)$. Keep 5 digits after decimal point for all final presented results.

- (1) Find the value of $\hat{\beta}_0$, the LSE and MLE for β_0 .

$$\hat{\beta}_0 = (1, 0, 0) [(X'X)^{-1}]_{(1,1)} = 2.34123.$$

- (2) Find $\text{var}(\hat{\beta}_0)$.

$$\text{var}(\hat{\beta}_0) = [(X'X)^{-1}]_{(1,1)} \sigma^2 = 0.11322 \sigma^2.$$

- (3) Find the value of standard error for $\hat{\beta}_0$.

$$S_{\hat{\beta}_0} = \sqrt{MSE [(X'X)^{-1}]_{(1,1)}} = 1.09673.$$

`proc reg; model y=x1 x2/i; run;`

2. With data in 1 consider model $y = \beta_1 x_1 + \beta_2 x_2 + \epsilon$, $\epsilon \sim N(0, \sigma^2)$.

$$\text{Let } x_0 = \begin{pmatrix} x_{01} \\ x_{02} \end{pmatrix} = \begin{pmatrix} 10 \\ 300 \end{pmatrix}.$$

Keep 5 digits after decimal point for all final presented results.

- (1) Find $\hat{y}(x_0)$.

$$\hat{y}(x_0) = x_0' \hat{\beta} = 10 \hat{\beta}_1 + 300 \hat{\beta}_2 = 10 \times 1.70790 + 300 \times 0.01612 = 21.91500$$

- (2) Find $S_{\hat{y}(x_0)}^2$ and $S_{\hat{y}(x_0)}$.

$$\begin{aligned} S_{\hat{y}(x_0)}^2 &= MSE x_0' (X'X)^{-1} x_0 \\ &= 12.26727 (10, 300) \begin{pmatrix} 0.00256898 & -0.000051145 \\ -0.000051145 & 0.011669 \times 10^{-4} \end{pmatrix} \begin{pmatrix} 10 \\ 300 \end{pmatrix} \\ &= 0.67530 = 0.82177^2. \end{aligned}$$

So $S_{\hat{y}(x_0)} = 0.82177$.

`proc reg; model y=x1 x2/noint i; run;`