**Stat763 HW03**

1. The usefulness of the model $y = \beta_0 + \beta_1 + \epsilon$ is confirmed by an F-test for the significance for regression based on a sample of $n = 20$. The following statistics are also obtained.

$$\widehat{\beta}_0 = 2628,\ S_{\widehat{\beta}_0} = 44,\ \widehat{\beta}_1 = -37, S_{\widehat{\beta}_1} = 3,\ \widehat{y}(10) = 2258 \text{ and } S_{\widehat{y}(10)} = 23.6.$$

Keep 3 digits after decimal point for all final computation results

(1) Find a 95% lower-sided CI for $\beta_0$.

$\widehat{\beta}_0 + t_{0.05}(18)S_{\widehat{\beta}_0} = 2628 + 1.734 \times 44 = 2704.296$. So

$$(-\infty,\ 2704.296) \text{ is a 95\% lower-sided CI for } \beta_0.$$

(2) Find a 95% upper-sided CI for $\beta_1$.

$\widehat{\beta}_1 - t_{0.05}(18)S_{\widehat{\beta}_1} = -37 - 1.734 \times 3 = -42.202$. So

$$(-42.202,\ \infty) \text{ is a 95\% upp-sided CI for } \beta_1.$$

(3) Find a 95% CI for $E[y(10)]$.

$\widehat{y}(10) \pm t_{0.025}(18)S_{\widehat{y}(10)} = 2258 \pm 2.101 \times 23.6 = 2258 \pm 49.584 = (2208.416,\ 2307.584)$ So

$$(2208.416,\ 2307.584) \text{ is a 95\% CI for } E(y(10)).$$

2. For $y = \beta x + \epsilon$, $\epsilon \sim N(0, \sigma^2)$, a sample of size $n = 8$ produced $\widehat{y}(0.65) = 0.4045$ and $S_{\widehat{y}(0.65)} = 0.0920$.
For final results of computations keep 4 digits after decimal points.

(1) Test $H_0 : E[y(0.65)] \geq 1$ vs $H_a : E[y(0.65)] < 1$ using rejection region with level 0.05.

$H_0 : E[y(0.65)] \geq 1$ versus $H_a : E[y(0.65)] < 1$
Test statistic $t = \frac{\widehat{y}(0.65)-1}{S_{\widehat{y}(0.65)}}$
Reject $H_0$ if $t < -1.895$ for $\alpha = 0.05$
$t = \frac{0.4045-1}{0.0920} = -6.4728$
Reject $H_0$.
We conclude that $E[y(0.65)] < 1$.

(2) Test $H_0: E[y(0.65)] \leq 2$ vs $H_a: E[y(0.65)] > 2$ using p-value.

$H_0: E[y(0.65)] \leq 2$ versus $H_a: E[y(0.65)] > 2$
Test statistic $t = \frac{\widehat{y}(0.65)-2}{S_{\widehat{y}(0.65)}}$
p-value: $P(t(n-1) > t_{ob})$
$t = \frac{0.4045-2}{0.0920} = -17.3424$
p-value: $P(t(7) > -17.3424) \approx 1$
Fail to reject $H_0$.
Data shows that $E[y(0.65)] \leq 2$.

(3) Perform t-test on $H_0: E[y(0.65)] = 0$ vs $H_a: E[y(0.65)] \neq 0$ using p-value.

$H_0: E[y(0.65)] = 0$ versus $H_a: E[y(0.65)] \neq 0$
Test statistic $t = \frac{\widehat{y}(0.65)}{S_{\widehat{y}(0.65)}}$
p-value: $2P(t(n-1) > |t_{ob}|)$
$t = \frac{0.4045}{0.0920} = 4.3967$
p-value: $2P(t(7) > 4.3967) = 2 \times 0.002 = 0.004$
Reject $H_0$.
Conclude that $E[y(0.65)] \neq 0$.