Stat763 HW01

1. A sample for regression produced

$$n = 8,\ \overline{x} = 1.375,\ \overline{y} = 3.125,\ Sxx = 19.875,\ Syy = 18.875,\ Sxy = 10.625.$$

For each required computation below, write out formula first followed by numerical results. Keep 3 digits after decimal point for final results.

(1) Find n, $\sum x$, $\sum y$, $\sum x^2$, $\sum y^2$ and $\sum xy$.

$n = 8$, $\sum x = n\overline{x} = 11$, $\sum y = n\overline{y} = 25$, $\sum x^2 = Sxx + \frac{(\sum x)^2}{n} = 35$,
$\sum y^2 = Syy + \frac{(\sum y)^2}{n} = 97$, $\sum xy = Sxy - \frac{(\sum x)(\sum y)}{n} = 45$.

(2) For model $y = \beta_0 + \beta_1 x + \epsilon$ with $\epsilon \sim N(0, \sigma^2)$ find $\widehat{\beta}_1$, $\widehat{\beta}_0$ and $\widehat{y}(1)$.

$\widehat{\beta}_1 = \frac{Sxy}{Sxx} = 0.535$, $\widehat{\beta}_0 = \overline{y} - \overline{x}\,\widehat{\beta}_1 = 2.390$
$\widehat{y}(1) = \widehat{\beta}_0 + \widehat{\beta}_1 \times 1 = 2.925$

(3) For the model with intercept find SSE, MSE, $S_{\widehat{\beta}_0}$, $S_{\widehat{\beta}_1}$ and $S_{\widehat{y}(1)}$.

$SSE = Syy - \frac{(Sxy)^2}{Sxx} = 13.195$, $MSE = \frac{SSE}{n-2} = 2.199$,
$S_{\widehat{\beta}_0} = \sqrt{MSE\left(\frac{1}{n} + \frac{\overline{x}^2}{Sxx}\right)} = 0.696$
$S_{\widehat{\beta}_1} = \sqrt{\frac{MSE}{Sxx}} = 0.333$, $S_{\widehat{y}(1)} = \sqrt{MSE\left(\frac{1}{n} + \frac{(1-\overline{x})^2}{Sxx}\right)} = 0.539$

2. A sample for the simple linear regression without intercept produced

$$n = 4,\ \sum x = 2,\ \sum y = 14,\ \sum x^2 = 6,\ \sum y^2 = 54,\ \sum xy = 4.$$

For the following required computation, write out formula and keep 4 digits after decimal point for you final results.

(1) Find $\widehat{\beta}$ and $\widehat{y}(2)$.

$\widehat{\beta} = \frac{\sum xy}{\sum x^2} = 0.6667$, $\widehat{y}(2) = \widehat{\beta} \times 2 = 1.3333$

(2) Find SSE, MSE, $S_{\widehat{\beta}}$ and $S_{\widehat{y}(2)}$.

$SSE = \sum y^2 - \frac{(\sum xy)^2}{\sum x^2} = 51.3333$, $MSE = \frac{SSE}{n-1} = 17.1111$
$S_{\widehat{\beta}} = \sqrt{MSE\frac{1}{Sxx}} = 1.6887$, $S_{\widehat{y}(2)} = \sqrt{MSE\,\frac{2^2}{\sum x^2}} = 3.3775$