Stat 763 Exam 2 Mar. 13, 2025

Name:

For all computations keep 4 digits after decimal points.

1. The following is part of SAS output for a regression model based on a sample of size 10.

```
X'X Inverse, Parameter Estimates, and SSE
 Variable      X1         X2         Y
    X1      0.02294   -0.01822    1.0668
    X2     -0.01822    0.01741    1.5028
    Y       1.0668     1.5028     238

 R-sq: 0.9000

          Analysis of Variance
       Source        DF        SS
       Model        _____   ________
       Error        _____   ________
       U. Total     _____   ________
```

(1) Write out the model the SAS output is for. (10 points)

$y = \beta_1 x_1 + \beta_2 x_2 + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$

(2) With $x_1 = 1$ and $x_2 = 1$, find estimated mean of response y. (15 points)

The mean of y when $x_1 = 1$ and $x_2 = 1$ is estimated by

$$\widehat{y} = \widehat{\beta}_1 + \widehat{\beta}_2 = 1.0668 + 1.5028 = 2.5696.$$

(3) Fill the blanks in the ANOVA table. (25 points)

From output SSE= 238 $\quad \begin{cases} \frac{SSM}{SSTO} = R^2 = 0.9 \\ SSTO - SSM = SSE = 238 \end{cases} \Rightarrow \begin{cases} SSM = 2142 \\ SSTO = 2380 \end{cases}$

Source	DF	SS
Model	2	2142
Error	8	238
U.Total	10	2380

2. With a sample of size 14 for a regression SAS produced Parameter Table below.

Variable	Estimate	S. E.	t	Pr>|t|
Intercept	1.3694	1.3218	1.04	0.3224
X1	0.5661	0.1821	3.11	0.0099
X2	-0.1725	0.1517	-1.14	0.2797

(1) Write out the model the SAS output is for. (10 points)

The model is $y = \beta_0 + \beta_1 x1 + \beta_2 x2 + \epsilon$, $\epsilon \sim N(0, \sigma^2)$.

(2) Find a 95% C. I. for the mean response when $x_1 = 0$ and $x_2 = 0$. (15 points)

$t_\alpha(DF)$	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
DF=10	2.764	2.228	1.812
DF=11	2.718	2.201	1.796
DF=12	2.681	2.179	1.782

$$\begin{aligned} \widehat{\beta}_0 \pm t_{\alpha/2}(n-p)S_{\widehat{\beta}_0} &= \widehat{\beta}_0 \pm t_{0.025}(11)S_{\widehat{\beta}_0} \\ &= 1.3694 \pm 2.201 \times 1.3218 = 1.3694 \pm 2.9093 \\ &= (-1.5399,\ 4.2787) \end{aligned}$$

is a 95% C. I. for the mean response when $x_1 = 0$ and $x_2 = 0$.

(3) Find the smallest confidence coefficient such that the lower-sided confidence interval for β_2 covers value 0. (25 points)

$H_0 : \beta_2 \geq 0$ versus $H_a : \beta_2 < 0$
Test Statistic: $t = \frac{\widehat{\beta}_2}{S_{\widehat{\beta}_2}}$
p-value: $P(t(n-c) < t_{ob})$
$t_{ob} = -1.14$
p-value: $P(t(11) < -1.14) = \frac{0.2797}{2} = 0.1399$

The smallest confidence coefficient for lower-sided confidence interval for β_2 that covers 0 is $1 - 0.1399 = 0.8601 = 86.01\%$.