# Stat 763 Exam 1 Feb. 13, 2025

Name:

0. $t_\alpha(k)$ table for this exam

$t_\alpha(k)$ table

| k | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|
| $\alpha = 0.05$ | 1.943 | 1.895 | 1.860 | 1.833 | 1.812 | 1.796 |
| $\alpha = 0.025$ | 2.447 | 2.365 | 2.306 | 2.262 | 2.228 | 2.201 |

For all final presented computation results keep 3 digits after decimal point.

1. For model $y = \beta x + \epsilon$, $\epsilon \sim N(0, \sigma^2)$ data $(x_i, y_i)$, $i = 1, ..., n$, produced

| $n$ | $\sum x$ | $\sum y$ | $\sum x^2$ | $\sum y^2$ | $\sum xy$ |
|---|---|---|---|---|---|
| 8 | 5.20 | 3.83 | 3.80 | 2.01 | 2.37 |

(1) Find the value of the least square estimator for $\beta$. (8 points)

$\widehat{\beta} = \frac{\sum xy}{\sum x^2} = \frac{2.37}{3.80} = 0.624.$
0.624 is the value of LSE for $\beta$.

(2) Find the value of an unbiased estimator for $\sigma^2$. (8 points)

$$\begin{aligned} MSE &= \frac{SSE}{n-1} = \frac{1}{n-1}\left[\sum y^2 - \frac{(\sum xy)^2}{\sum x^2}\right] \\ &= \frac{1}{7}\left(2.01 - \frac{2.37^2}{3.80}\right) = 0.076. \end{aligned}$$

is the value of an unbiased estimator for $\sigma^2$.

(3) Find the value of standard error for $\widehat{\beta}$. (8 points)

$S^2_{\widehat{\beta}} = \frac{MSE}{\sum x^2} = \frac{0.076}{3.80} = 0.02 = 0.141^2$
The standard error for $\widehat{\beta}$ is $S_{\widehat{\beta}} = 0.141$.

(4) Fill out ANOVA table (26 points)
For $p$-value, either leave with a formula or put in your estimated approximate value.

| Source | SS | DF | MS | F | p |
|---|---|---|---|---|---|
| Model | 1.478 | 1 | 1.478 | 19.447 | $\approx 0$ |
| Error | 0.532 | 7 | 0.076 | | |
| U.Total | 2.010 | 8 | | | |

2. For model $y \sim N(\beta_0 + \beta_1 x, \sigma^2)$ data $(x_i,\, y_i)$, $i = 1, ..., n$, produced

| $n$ | $\overline{x}$ | $\overline{y}$ | Sxx | Syy | Sxy |
|---|---|---|---|---|---|
| 12 | 7.72 | 9.50 | 199.80 | 140.78 | 136.52 |

(1) Is the model good for the data? Support your conclusion by a t-test. Write your report using rejection region at the level 0.05. (25 points)

$H_0 : \beta_1 = 0$ versus $H_a : \beta_1 \neq 0$

Test Statistic: $t = \frac{\widehat{\beta}_1}{S_{\widehat{\beta}_1}}$

Reject $H_0$ if $t < -2.228$ or $t > 2.228$ for $\alpha = 0.05$

$\widehat{\beta}_1 = \frac{Sxy}{Sxx} = \frac{136.52}{199.80} = 0.683$

$S^2_{\widehat{\beta}_1} = \frac{SSE}{(n-2)\,Sxx} = \frac{47.498}{10 \times 199.80} = 0.02377 = 0.154^2$

$t = \frac{0.683}{0.154} = 4.435$

Reject $H_0$. The model is good for the data.

(2) Find a 95% confidence interval for $E(y)$ when $x = 0$. (25 points)

$\widehat{\beta}_0 = \overline{y} - \overline{x}\,\frac{Sxy}{Sxx} = 9.50 - 7.72 \cdot \frac{136.52}{199.80} = 4.225$

$SSE = Syy - \frac{(Sxy)^2}{Sxx} = 140.78 - \frac{136.52^2}{199.80} = 47.498$

$S^2_{\widehat{\beta}_0} = \frac{SSE}{n-2}\left(\frac{1}{n} + \frac{\overline{x}^2}{Sxx}\right) = \frac{47.498}{10}\left(\frac{1}{12} + \frac{7.72^2}{199.80}\right) = 1.8126 = 1.346^2$

$\widehat{\beta}_0 \pm t_{0.025}(10) S_{\widehat{\beta}_0} = 4.225 \pm 2.228 \times 1.346 = 4.225 \pm 2.999 = (1.226, 7.224)$

is a 95% confidence interval for $\beta_0$, $E(y)$ when $x = 0$.