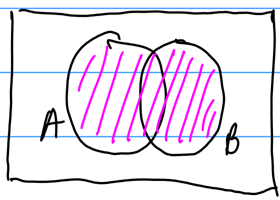
M321 1/29/20

§1.2 Sets

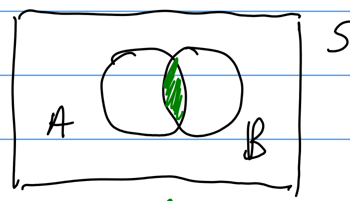
Let A and B represent two sets.

<u>Union</u>: $A \cup B = \{x \mid x \in A \lor x \in B\}$ Disjunction

<u>Intersection</u>: $A \cap B = \{x \mid x \in A \land x \in B\}$ Conjunction



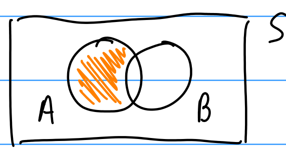
$A \cup B$



$A \cap B$

The <u>difference</u> of A and B is the set

$$A \neg B = A \setminus B = \{x \mid x \in A \land x \notin B\}$$



$A \neg B = A \setminus B$

<u>Ex</u>. $\alpha = \{W, I, C, H, T, A\}$ $\eta = \{K, A, N, S\}$

$\alpha \cap \eta = \{A\}$

$\alpha \cup \eta = \{W, I, C, H, T, A, K, N, S\}$

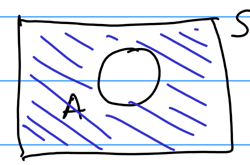
$\alpha \neg \eta = \alpha \setminus \eta = \{W, I, C, H, T\}$

$\eta \neg \alpha = \eta \setminus \alpha = \{K, N, S\}$

"Exclusive Or"

$(\alpha \cup \eta) \neg (\alpha \cap \eta) = \{W, I, C, H, T, K, N, S\}$

Look at Table 1.2.20.



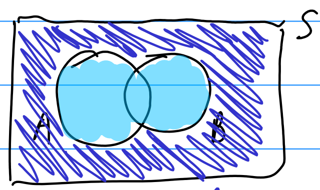
A^c

Defn. Let A be a set in some universal set S.
The complement of A (wrt S) is the set

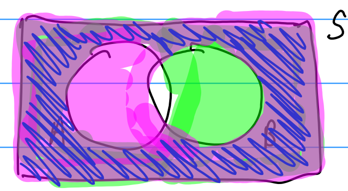
$$A^c = \bar{A} = S \setminus A$$

Examples: $A^c \cap A = \emptyset$ ✓

DeMorgan's Laws: $(A \cup B)^c = A^c \cap B^c$



$(A \cup B)^c$



$A^c \cap B^c$

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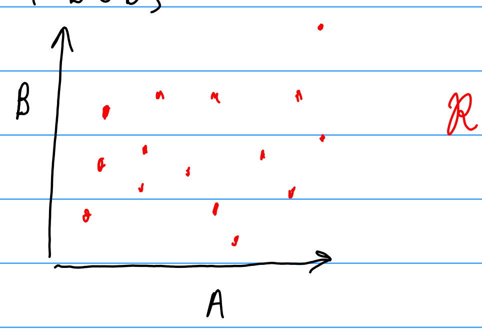
§1.3 Functions and Relations

Let A and B be sets. The (Cartesian) product of A with B is the set

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

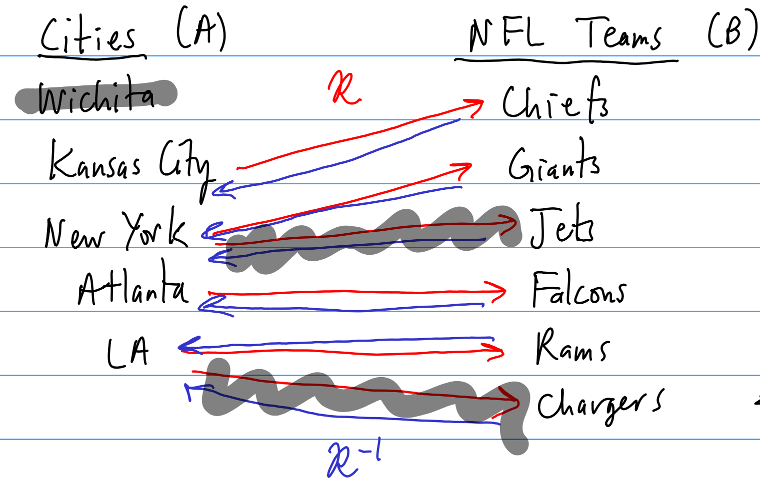


Defn. A relation between the sets A and B (from A to B) is a subset of the product set, $R \subseteq A \times B$.

$$R \subseteq \{(x,y) \mid x \in A \wedge y \in B\}$$

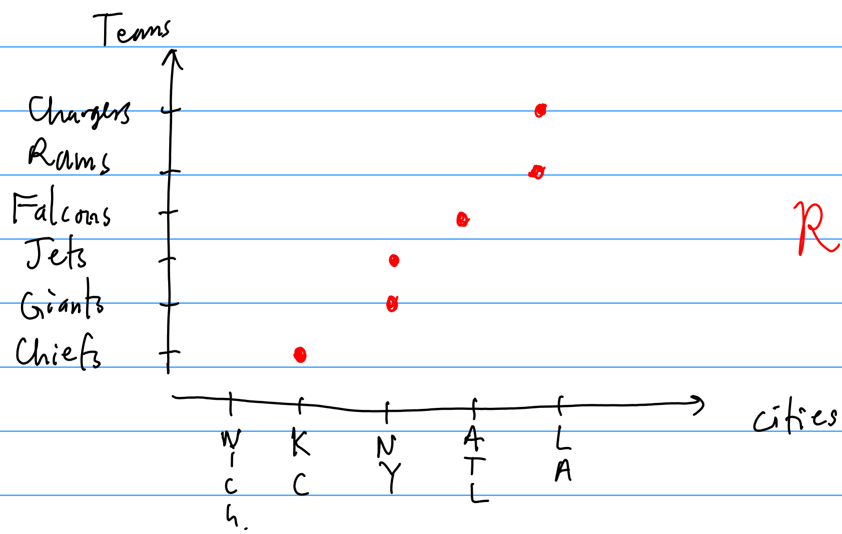
If $(x,y) \in \mathcal{R}$, we write $x \mathcal{R} y$ or $x \sim y$ to mean x is related to y by $\mathcal{R}$.

Ex.



Removing these arrows was a choice w/ consequences! Two different choices give two different functions.

A is the domain and B is the range.



Def'n. Given a relation $\mathcal{R} \subseteq A \times B$, the inverse relation to $\mathcal{R}$ is the relation $\mathcal{R}^{-1} \subseteq B \times A$ defined by

$$\mathcal{R}^{-1} = \{ (y,x) \mid (x,y) \in \mathcal{R} \}$$

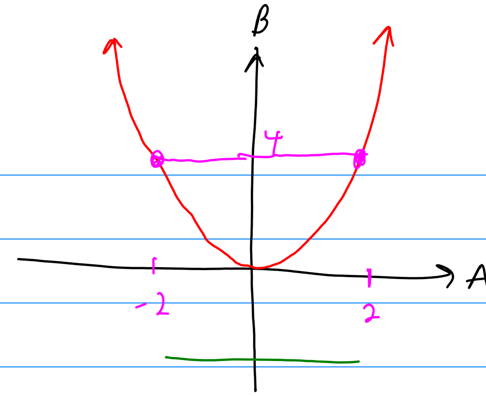
Def'n. A function is a relation from A to B satisfying: for every $x \in A$ there is exactly one $y \in B$ such that $x \mathcal{R} y$.

"every $x \in A$ has exactly one arrow coming out"

Ex. $x R y$ iff $y = x^2$

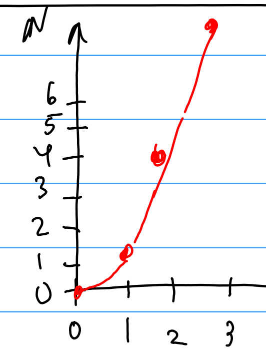
$\begin{cases} A = \mathbb{R} \\ B = \mathbb{R} \end{cases}$

Not Injective
Not surj.



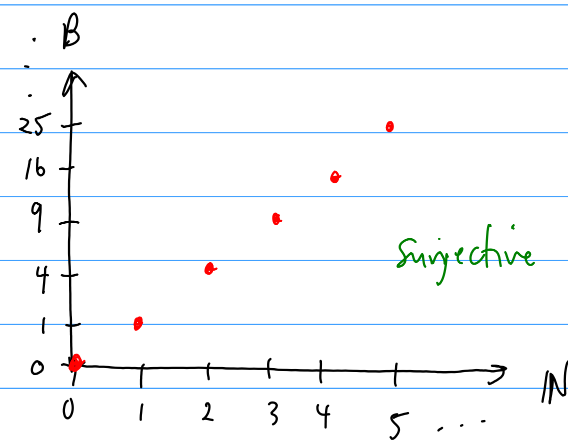
Ex. $x R y \leftrightarrow y = x^2$

$A = \mathbb{N}$
$B \subseteq \mathbb{N}$
$B = \{0, 1, 4, 9, 16, 25, \ldots\}$



Not Surj / Yes Inj

Injective!



Surjective

<u>Def'n.</u> A function $f : A \to B$ is said to be <u>injective</u> if:
if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Also called "one-to-one"
<u>Arrows:</u> Every point in B w/ at least one arrow going in has exactly one arrow going in.

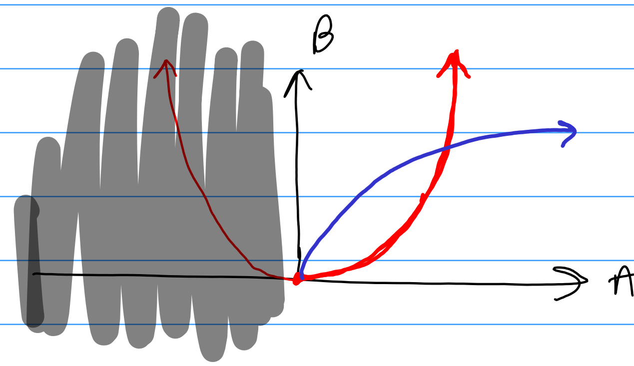
<u>Def'n.</u> A <u>function</u> is <u>surjective</u> if: For every $y \in B$ there exists an $x \in A$ such that $y = f(x)$.
"onto" Arrows: "every pt in B has at least one arrow coming in"

Def'n. A function is bijective iff it is both injective and surjective.

Thm. A function $f: A \to B$ has an inverse function iff f is bijective. We write $f^{-1}: B \to A$.

Ex. $xRy \Leftrightarrow y = x^2$

$$\left.\begin{array}{l} A = \mathbb{R}_0^+ \\ B = \mathbb{R}_0^+ \end{array}\right\} \text{bijective}$$



Inverse function is $xRy \leftrightarrow y = \sqrt{x}$