M321 1/27/20

§1.2 - Sets

Def'n. A set is an unordered collection of objects, called elements or members of the set.

We write A, B, C, S to denote the set
and a, b, c, s to denote the elements.
We write $a \in B$ to mean that "a is an element of B"

Ex.
a is not an element of C can be written $\neg(a \in C)$ or $a \notin C$

Examples of Number sets:

$\mathbb{R}$ = the set of all real numbers
or $\mathbb{R} = (-\infty, +\infty)$
We can define sub-intervals: ex. $(-5, 4] = \{x \mid -5 < x \leq 4\}$

$\mathbb{R}^+ = \{x \mid x \in \mathbb{R} \text{ and } x > 0\} = \{x \in \mathbb{R} \mid x > 0\}$

$\mathbb{R}^+_0 = \{x \in \mathbb{R} \mid x \geq 0\}$

Def'n. The empty set is the set with no elements.
We write $\emptyset = \{\ \}$

$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$ Natural numbers
$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ Integers

so $\mathbb{Z}^+_0 = \{z \in \mathbb{Z} \mid z \geq 0\} = \{0, 1, 2, 3, \ldots\} = \mathbb{N}$
$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ Positive Integers

$\mathbb{Q} = \{ \frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}^+ \}$ Rational Numbers

$\mathbb{C} = \{ a+ib \mid i = \sqrt{-1},\ a,b \in \mathbb{R} \}$ Complex Numbers

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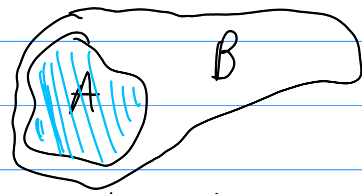
Def'n. Let A and B be two sets. We say that A is a subset of B iff all of the elements in A are also in B.



Two sets are equal iff they have exactly the same members. The two sets are subsets of each other.

Notation: A is a subset of B : $A \subseteq B$ or $A \subset B$

A is equal to B: $A = B \leftrightarrow ((A \subseteq B) \wedge (B \subseteq A))$

Def'n. For any set S, there are two trivial subsets:

1. $\emptyset \subseteq S$ (smallest)
2. $S \subseteq S$ (largest)

Def'n. Any subset of S that is not trivial is called proper.

Def'n. The cardinality of a set S is the "number" of elements in the set. We write $|S|$, $\#(S)$

Ex. $S = \{a,b,c\}$ $|S| = 3$ has 8 total subsets.

List the subsets of S:

0	$\emptyset$
1	$\{a\}, \{b\}, \{c\}$ Singletons
2	$\{a,b\}, \{a,c\}, \{b,c\}$ Doubletons
3	$\{a,b,c\}$

Def'n. The <u>power set</u> of S is the set whose elements are all of the subsets of S, denoted by $\mathcal{P}(S)$.

Theorem. The cardinality of a power set is $|\mathcal{P}(S)| = 2^{|S|}$.

Ex. $S = \{A, B, C, D, E, \ldots, X, Y, Z\}$ $\quad |S| = 26$

How many subsets? 2^{26} ~~too big to write down~~

$= 67108864$

Ex. $S = \{a, b, c\}$ $\quad |S| = 3$

$|\mathcal{P}(\mathcal{P}(S))|$

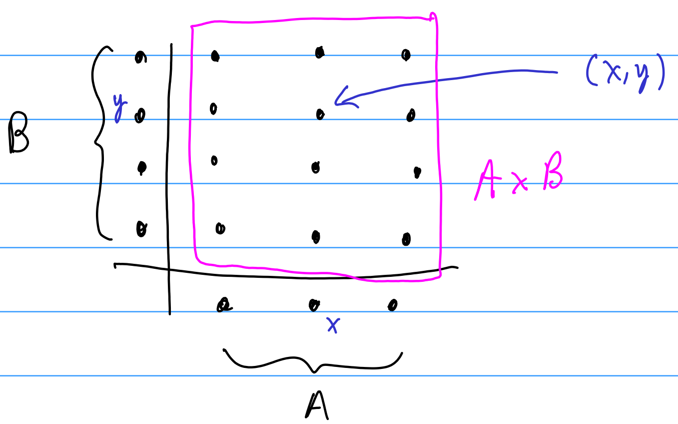
$\mathcal{P}(S) = \Big\{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\} \Big\}$ $\quad |\mathcal{P}(S)| = 8$

$\mathcal{P}(\mathcal{P}(S))$: $\left.\begin{array}{l} \emptyset \\ \{\emptyset\}, \{\{a\}\}, \ldots \end{array}\right\}$ $\quad |\mathcal{P}(\mathcal{P}(S))| = 2^{|\mathcal{P}(S)|} = 2^8 = 256$

Operations on sets:

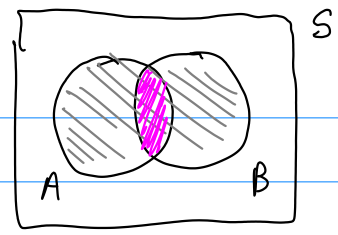
Def'n. The (<u>Cartesian</u>) <u>product</u> of two sets A and B is the set constructed as

$$A \times B = \{(a,b) \mid a \in A, b \in B\} \qquad \text{ordered pairs}$$



Def'n. Let A and B sets. The <u>union</u> of A and B is the set defined by $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Venn Diagram:



$S \leftarrow$ Universal set

$A \cup B$

$A \cap B$

Defn. The intersection of A and B is the set

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$