M321 22 Jan 2020

§1.1 - Propositional Logic

Defn. A proposition is a statement that is either true or false, but not both.

Ex. p = "this course is discrete math" — yes, a prop

Chiefs are the best — subjective, so no.

Wichita is the capital of Kansas — False, yes a prop.

$1-3=4$ — Prop.

Operations w/ Propositions

other sym: ~ ! not

1. Negation: The negation of a proposition p is the proposition denoted by $\neg p$ which has the opposite truth value of p.

Ex. p = "This course is discrete math"

$\neg p$ = "It is not the case that this course is discrete math."

or "This course is not discrete math."

Ex. p = "$1-3=4$"

$\neg p$ = "$1-3\neq 4$"

2. Conjunction: Let p and q be propositions. The conjunction of p and q is a proposition, denoted by $p\wedge q$, read as "p and q."

Truth Table:

p	q	$p\wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3. <u>Disjunction</u>: Let p and q be propositions. This <u>disjunction</u> of p and q is the proposition denoted by $p \vee q$, and read as "p or q, or both." (Inclusive!)

Truth Table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4. <u>Conditional</u>: Let p and q be props. The <u>conditional</u> of p and q is denoted by $p \to q$ read as

"if p, then q"

"p implies q"

"q if p"

"p only if q"

Truth Table:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

5. <u>Biconditional</u>: Let p and q be propositions. The <u>biconditional</u> of p and q is denoted by $p \leftrightarrow q$, and is defined to be

$$p \leftrightarrow q = \big((p \to q) \wedge (q \to p) \big)$$

"p if and only if q"

"p iff q"

Truth Table:

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Def'n's. Consider the conditional $p \to q$.

1. The converse is $q \to p$
2. The inverse is $\neg p \to \neg q$
3. The contrapositive is $\neg q \to \neg p$

Def'n. A proposition is said to be a tautology if it is always T.
A contradiction is always F.
Contradictions and tautologies are negations of one another.
Two propositions are logically equivalent if their biconditional is a tautology. $(p \leftrightarrow q)$

Ex. Show via truth table that $p \to q$ is logically equivalent to its contrapositive.

$$(p \to q) \equiv (\neg q \to \neg p)$$

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology!

Ex. $(p \wedge q) \to r$

p	q	r	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

§1.1 JH's notes.