M321 - Discrete Structures I 21 Jan 2020

Propositional Logic

Def'n. A proposition is a statement that is either true or false, but not both.

Ex. My car is red. — Yes - a proposition
I like my car. — Not a proposition
$1-3=4$ — Yes, it is a proposition.
What are you doing? — No.

Operations on propositions

Def'n. Let p be a logical proposition. The negation of p is the proposition denoted by $\neg p$ having the opposite truth value of p.

Ex. $p =$ "Wichita is the capital of Kansas"
$\neg p =$ "Wichita is not the capital of Kansas"

$q =$ "$1-3=4$"
$\neg q =$ "$1-3 \neq 4$"

Def'n. Let p and q be two propositions.
The conjunction of p and q is the proposition $p \wedge q$ read as "p and q".

Truth Tables:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The disjunction of p and q is the proposition $p \vee q$, read as "p or q". (or both)

Truth Table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Defn. Let p and q be propositions. The conditional statement $p \to q$ is read as "If p, then q."

or "q if p" "p implies q"

TT:

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q = (p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Defn. The biconditional, $p \leftrightarrow q$, is read as "p if and only if q" is logical equivalent to the statement

$$(p \to q) \wedge (q \to p)$$

Given a conditional: $p \to q$

The converse is $q \to p$

The inverse is $\neg p \to \neg q$

The contrapositive is $\neg q \to \neg p$

Ex. p = Justin's jacket is black

q = this is proposition

1. $\neg p \vee q$ = "Justin's jacket is not black or this is a proposition."

2. $p \to q$ = "If J's jacket is black, then this is a prop."

3. $\neg q \to \neg p$ = "If this is not a proposition, then J's jacket is not black."

Defn. A statement is a tautology if its truth value is always T.
A contradiction is a statement that's truth value is always F.

The negation of a tautology is a contradiction, and vice versa.

Defn. Two propositions p and q are logically equivalent if their biconditional is a tautology: $p \leftrightarrow q$ is always T.

Ex. Construct a truth table to show that $p \to q$ is logically equivalent to its contrapositive: $\neg q \to \neg p$

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology

Ex. Construct a truth table for $(p \wedge q) \to \neg r$

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \to \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	F	F	T
F	F	F	F	T	T