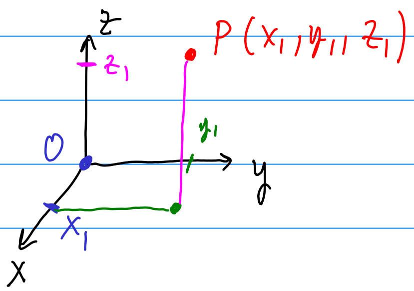
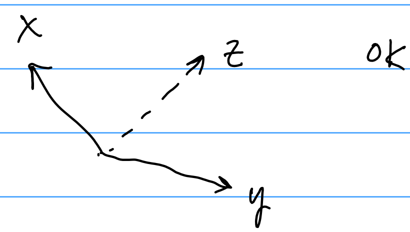
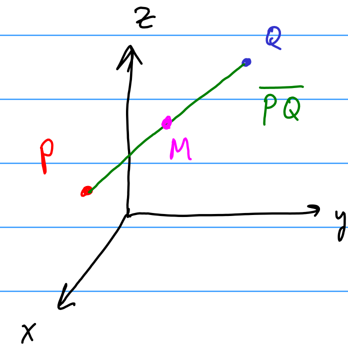
§12.1 - 3D Coordinates





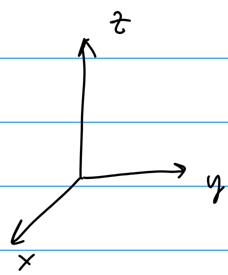
<u>Ex.</u> Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

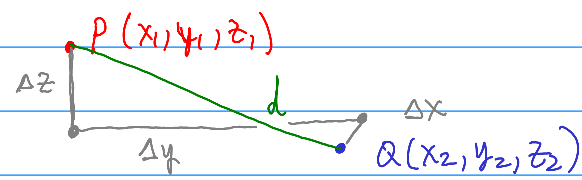
<u>QI.</u> What is the midpoint of $\overline{PQ}$.



The <u>midpoint formula</u> is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$





<u>Pythagorean</u> <u>Thm</u>:

$$d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

Proof?

The <u>distance</u> between P and Q is given by

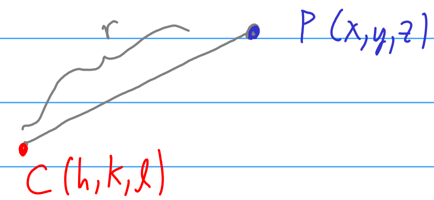
$$d(P, Q) = d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

or

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a sphere?

Def'n. A <u>sphere</u> is the set of all points in space that are equidistant from a fixed point.
The fixed point is the <u>center</u>.
The fixed distance is the <u>radius</u>.



by distance formula, all points $P(x,y,z)$ that lie on the sphere obey the equation

$$d(P,C) = r$$

but $d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$

so, putting this together, we get

$$\boxed{(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2}$$

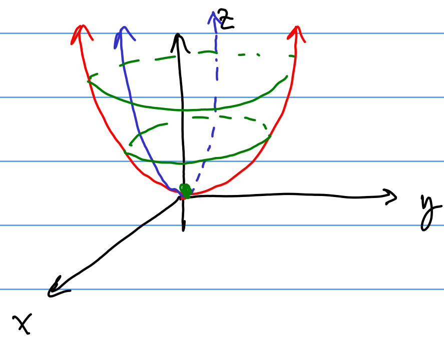
Ex. $C(\underset{h}{2}, \underset{k}{-15}, \underset{l}{\pi})$, $r = 4$ $\longrightarrow$ $(x-2)^2 + (y+15)^2 + (z-\pi)^2 = 16$

Ex. $x^2+y^2+z^2+4x-6y+2z+6=0$ Is it a sphere? Yes!

$$x^2+4x+\underline{4} + y^2-6y+\underline{9} + z^2+2z+1 = -6+4+9+1$$

$$(x+2)^2+(y-3)^2+(z+1)^2 = 8 \xrightarrow{\checkmark} C(-2,3,-1),\ r=\sqrt{8}$$

Ex. $z = x^2 + y^2$ How does the graph look?



$x = 0$: $z = y^2$

$y = 0$: $z = x^2$

$z = 0$: $0 = x^2 + y^2$