Riemann Surfaces
Lecture 6

We're finally ready to define a Riemann surface! First, a couple of preliminary definitions.

Definition holomorphic maps.

Definition 2.1.1 surface.

Definition 2.1.2 Riemann surface.

Remarks

More remarks

Example 2.1.1 Trivial examples

Example 2.1.2 Riemann sphere

Example 2.1.2, continued

Example 2.1.3 $\mathbb{T}^2$

Definition (sub/super) harmonic functions

Lemma 2.2.1 *On a compact Riemann surface S, every subharmonic function (hence also every harmonic or holomorphic function) is constant.*

Sketch of proof

Lemma 2.2.2 *Let S be a simply connected surface, and $F : S \to \mathbb{C}$ a continuous function, nowhere vanishing on S. Then* $\log(F)$ *can be defined on S. That is, there exists a continuous function f on S with $e^f = F$.*

Sketch of proof

Definition harmonic conjugates

Lemma 2.2.3 *Let S be a simply connected Riemann surface, and $u : S \to \mathbb{R}$ a harmonic function. Then there exists a harmonic conjugate to u on the whole of S.*

Sketch of proof