A pants decomp. is ordered by ordering the ends, partition curves, and regions, and the banks: $B_1, \dots, B_n$

$C_1, \dots, C_d$

$\sigma_1, \dots, \sigma_a$

b_{i1}, b_{i2}, b_{i3} of σ_i

Let the banks of the j^{th} partition curve C_j $(j=1,\dots,d)$ be $b_{k_j\mu_j}$ and $b_{l_j\nu_j}$ with either $k_j < l_j$ or $k_j = l_j$ and $\mu_j < \nu_j$. Then

$$k_1 = 1 \qquad l_1 = 2 \qquad \text{(consistency conditions)}$$

$$k_r \in \{1, \dots, r\} \qquad l_r = r+1 \qquad (r = 2, \dots, a-1)$$

Assume now that X is equipped w/ a consistently ordered maximal ~~decomp~~ partition (pants decomp). (For type $(0,3)$ there are no partition curves. An ordered partition is an ordering of the ends of X.)

A homeo f of X onto a Riemann surface will be called standardized wrt a given partition iff f maps every partition curve onto a geodesic.

Prop 7.1 Every $[f] \in \mathcal{T}(X)$ contains a standardized homeomorphism.

Let $[f] \in \mathcal{T}(X)$ w/ f standardized. This condition does not determine f uniquely but the geodesics $f(C_j)$, ends $f(B_i)$, regions $f(\sigma_k)$, and banks $f(b_{kr})$ depend only on $[f]$, so $f(X)$ is equipped w/ a consistently ordered maximal partition w/ geodesic partition curves.

Let σ be a partition region, s $(s = 1, 2, \text{or } 3)$ of whose banks are banks of partition curves; we call them bounding curves of σ. It is convenient to introduce the domain $f(\sigma)_{ext}$ which is the union of $f(\sigma)$, the images $f(C)$ of the s bounding partition curves C of σ and of s funnels attached to the geodesic Jordan curves $f(C)$. If $s=3$, then $f(\sigma)_{ext}$ is the Nielsen extension of $f(\sigma)$.

On every partition curve $f(C_j)$, $j=1,\dots,d$, there are two (not necessarily distinct) distinguished points "belonging" to the two banks of $f(C_j)$ and defined as follows.

Let b be a bank of C_j, σ_i the unique partition region on X containing b, and $f(\sigma_i)^{ext}$ the domain defined above. The ordering of the banks of σ_i induces an ordering of the ends of $f(\sigma_i)^{ext}$; $f(C_j)$ is an outer loop on this domain. The distinguished point on $f(C_j)$ belonging to the bank $f(b)$ is the first distinguished point of $f(C_j)$ from §6.

Rmk. The hyperbolic metrics on $f(X)$ and the restriction of the one on $f(\sigma_i)^{ext}$ to $f(\sigma_i)$ coincide.

Let $[f] \in \mathcal{F}(X)$ w/ f standardized. For every end B_k of X, let $2\pi\rho_k$ be the length of the outer loop of $f(X)$ defining $f(B_k)$, w/ $\rho_k = 0$ if $f(B_k)$ is a puncture. For every partition curve C_j, let $2\pi r_j$ be the length of the loop $f(C_j)$, and let θ_j denote any real number w/ the following property: if we approach a distinguished point P of $f(C_j)$ along a geodesic segment in the bank belonging to P, then turn to the right or left according to whether $\theta_j \geq 0$ or $\theta_j \leq 0$, and proceed the distance $r_j|\theta_j|$ along $f(C_j)$, we reach the other distinguished point Q on $f(C_j)$.
θ_j is determined modulo 2π w/ $\theta_j \equiv 0 \pmod{2\pi}$ iff $P=Q$.

It is clear that the numbers $r_j e^{i\theta_j}$, $j=1,\dots,d$, and ρ_k, $k=1,\dots,n$ depend only on $[f]$, so that we can define a map

$$\Phi : \mathcal{F}(X) \longrightarrow (\mathbb{C}^*)^d \times (\mathbb{R}_+ \cup \{0\})^n$$

by setting $\Phi([f]) = (r_1 e^{i\theta_1}, r_2 e^{i\theta_2}, \dots, r_d e^{i\theta_d}, \rho_1, \dots, \rho_n)$. Φ is the reduced Fenchel-Nielsen map.

Lemma 7.1 The reduced Fenchel-Nielsen map is a continuous surjection.

Rmk. The map f, or point $[f]$, is not uniquely determined by $\Phi([f])$.

Proof. Continuity follows from the numbers $r_j e^{i\theta_j}$ and ρ_k being continuous functions of $x_{[f]}(g_1), \dots, x_{[f]}(g_N)$, the generators defined in §3.

Let $v = (z_1, \dots, z_d, \rho_1, \dots, \rho_n) \in (\mathbb{C}^*)^d \times (\mathbb{R}_+ \cup \{0\})^n$. Using the numbers $r_j = |z_j|$ and ρ_k we construct the triply connected domains S_i, $i = 1, \dots, a$ which would have to be the domains $f(\sigma_i)$ if there were a homeomorphism f of X onto a Riemann surface $S = f(X)$ w/

$$\Phi([f]) = v.$$

Using the arguments of the numbers $z_1, \dots, z_d$ we can construct the surface S and a map $f: X \to S$ satisfying the defn. $\square$

Rmk. The construction of S consists of identifying d pairs of ideal boundary curves of the S_i; the curves to be identified may belong to the same or different S_i. The identification is by an isometry in the relevant hyperbolic metrics and is uniquely determined by ~~the~~ the argument of the relevant z_j.