§1.2 – Mention that David Hilbert revised Euclid's axioms, arranging them in groups:

- Connection
- Order
- Congruence
- Parallels
- Continuity

We will use this section mostly only for future reference. It includes definitions of some basic geometric objects.

For example,

<u>Defn</u>. 1.2.2 – Two lines, a line and a segment, or two segments, are said to <u>intersect</u> if there is a point that is on both of them.

Please read the 8 definitions in this section, if not the rest of the chapter.

<u>RE</u> – Draw pictures to illustrate each definition in this section (for which a picture is appropriate).

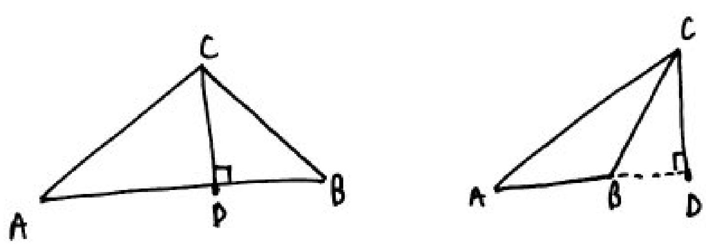
<u>§1.3 – Some Prereq. Material</u>

Lots more definitions in this section. You will <u>need</u> to know these.

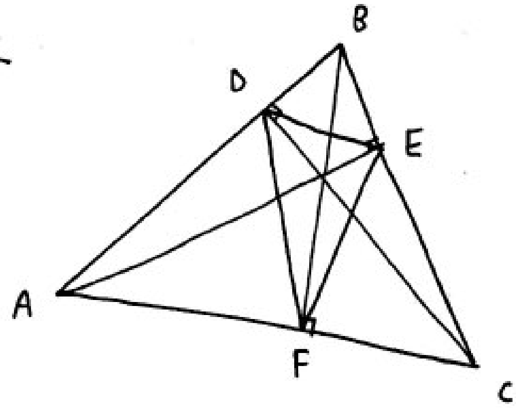
We will discuss a few of them, specifically related to triangles.

<u>Defn</u>. 1.3.5 –

<u>Altitudes</u>:

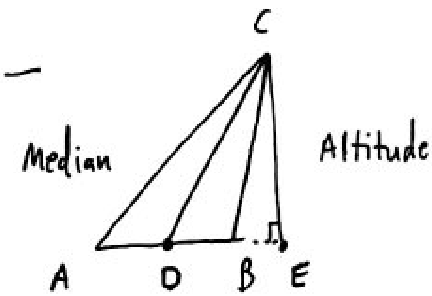


Defn. 1.3.6 —



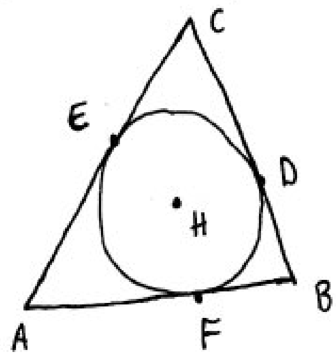
ΔDEF is the orthic triangle of ΔABC.

Defn. 1.3.7 —



CD is the median of ΔABC
CE is the altitude

Defn. 1.3.9 – A circle is inscribed in a polygon iff each side of the sides of the polygon is tangent to the circle.

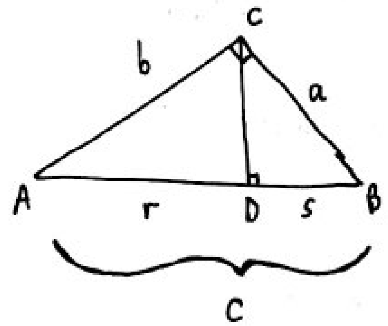


"Incircle"

H is the incenter.

Now, some theorems.

<u>Thm 1.3.12</u> - The altitude on the hypotenuse of a right triangle forms two triangles that are similar to the given triangle, and similar to each other.



<u>RE</u>. Prove using congruent angles.

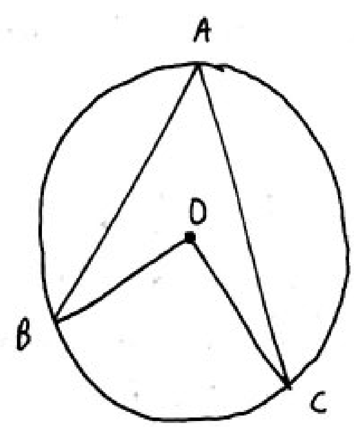
<u>Thm 1.3.13</u> - <u>Pythagorean Theorem</u>. If ΔABC is right with right angle at C, then $a^2+b^2=c^2$.

<u>Proof</u>. By Thm. 1.3.12, $\Delta ABC \sim \Delta ACD \sim \Delta CBD$. Therefore $\frac{a}{s}=\frac{c}{a}$ and $\frac{b}{r}=\frac{c}{b}$. Thus $b^2=cr$ and $a^2=cs$. Adding these expressions,

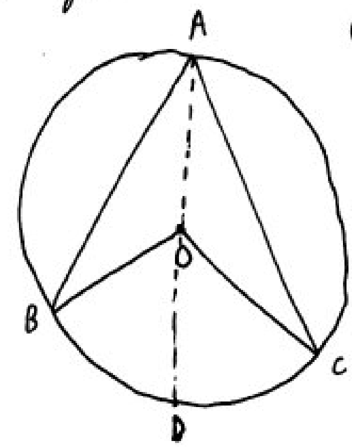
$$a^2+b^2=cr+cs=c(r+s)=c^2. \qquad \square$$

<u>Thm. 1.3.14</u> - <u>Star Trek Lemma</u>. The measure of an inscribed angle is half of the angular measure of the arc it subtends.

<u>Acute</u>:



Proof. Suppose $\measuredangle BAC$ is acute and O lies inside the angle.



Draw a diameter AOD.

Then $AO = OB = OC = OD$, so that ΔAOB is isosceles. Hence, $\measuredangle BAO = \measuredangle OBA$.

$\measuredangle BOD = \measuredangle OBA + \measuredangle BAO = 2\measuredangle BAO$.

Similary, one may show that

$\measuredangle DOC = 2\measuredangle OAC$.

Combining these, we obtain

$\measuredangle BOC = 2\measuredangle BAO + 2\measuredangle OAC = 2\measuredangle BAC$.

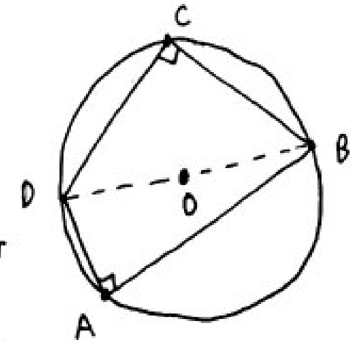
This proves the theorem when $\measuredangle BAC$ is acute and O is inside the angle.

The remaining cases are similar (RE). □

Defn. 1.3.11 – A quadrilateral whose vertices lie on a circle is called a cyclic quadrilateral.

Thm. 1.3.15 – If two opposite angles of a quadrilateral are both right, then the quadrilateral is cyclic.

Proof – (sketch)



Since $\measuredangle C$ is right, $\overline{DOB}$ must be a diameter of the circle through D, B, C.
Since $\measuredangle A$ is right, $\overline{DOB}$ must be a diameter of the circle through D, B, A.
Circles $ODBA$ and $ODBC$ share a diameter, so they must coincide. □