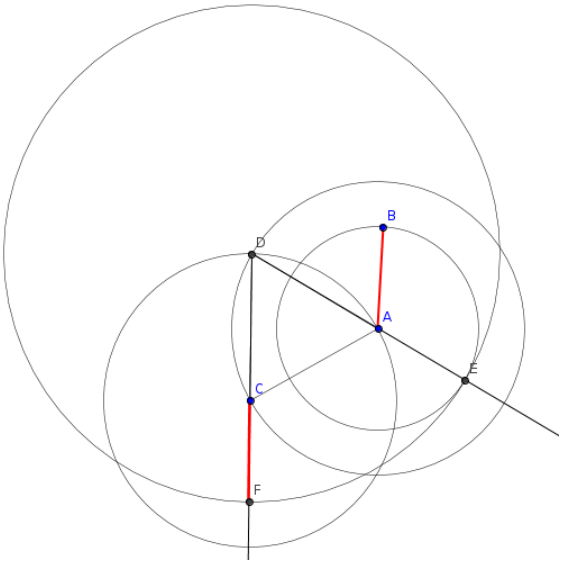
Sample Written Homework

Math 621: Elementary Geometry

Exercise. Use a compass and straightedge to construct a line segment equal in length to a given line segment, and based at a given point. Then use elementary geometric properties to prove that the constructed segment is indeed equal in length to the given segment.

Construction. This construction was made using GeoGebra.



Method of Construction. Suppose $\overline{AB}$ is the given line segment and C the given point. Draw the line segment $\overline{AC}$, then construct the equilateral triangle on $\overline{AC}$ by using the method of the proof of Proposition 1.1.1. Denote the third vertex of this triangle by D.

Next, construct the circle centered at A with radius $\overline{AB}$. Extend the line segment $\overline{DA}$ and denote the intersection of this ray with the last constructed circle as E.

Now construct the circle centered at D with radius $\overline{DE}$. Extend the line segment $\overline{DC}$ and denote the intersection of this ray with the last constructed circle as F.

The line segment $\overline{CF}$ is equal in length to $\overline{AB}$ and based at C.

Proof. We must prove the claim that the line segment $\overline{CF}$ is equal in length to $\overline{AB}$.

We first note that the segments $\overline{DE}$ and $\overline{DF}$ are equal in length because they are radii of the same circle. Further, the sub-segments $\overline{DA}$ and $\overline{DC}$ are equal in length as they are legs of an equilateral triangle. Therefore the remainder segments $\overline{CF}$ and $\overline{AE}$ are also equal in length.

Now, $\overline{AE}$ and $\overline{AB}$ are equal in length as they are radii of the same circle. Thus, since $\overline{CF}$ and $\overline{AB}$ are each equal in length to $\overline{AE}$, it follows that they are equal in length to each other. □