Differential Equations: Project 7

Due: Monday, 22 July 2013

In this project you will extend the methods of chapter 3 to higher order linear differential equations with constant coefficients.

1. Consider the homogeneous equation

$$y''' + 8y = 0$$

a.) Write down the characteristic equation and solve for r (there will be 3 solutions).

b.) For r_1, r_2, and r_3 write the corresponding solutions y_1, y_2, and y_3 to the DE just as you did in chapter 3.

c.) Show that the solutions y_1, y_2, and y_3 are linearly independent (*i.e.*, different) by taking the Wronskian $W(y_1, y_2, y_3)$.

d.) The general solution of the DE is $y = c_1y_1 + c_2y_2 + c_3y_3$.

2. Use the method of problem 1 to find the general solution of the differential equation

$$y^{(4)} + 6y''' + 12y'' + 10y' + 3y = 0.$$

3. Write the general form of the solution to the DE

$$y^{(4)} + 6y''' + 12y'' + 10y' + 3y = 3e^{-t} + e^{-3t} + 2\cos t.$$

Do not solve for the unknown coefficients.

4. Determine the general solution of the DE

$$y''' - y'' - y' + y = 2e^{-t} + 3$$

This time solve for the coefficient(s) explicitly.

5. Solve the IVP

$$y''' + 4y' = t; \quad y(0) = y'(0) = 0, \quad y''(0) = 1.$$