

DE: Project 4: Solutions

1. & 2. Check online .xls file.

3. $\varphi_0(t) = 0$

$$\varphi_1 = \mathcal{P}\varphi_0 = \int_0^t 1 - 0^3 ds = t$$

$$\varphi_2 = \mathcal{P}\varphi_1 = \int_0^t 1 - s^3 ds = s - \frac{1}{4}s^4 \Big|_0^t = t - \frac{1}{4}t^4$$

$$\varphi_3 = \mathcal{P}\varphi_2 = \int_0^t 1 - \left(s - \frac{1}{4}s^4\right)^3 ds = \int_0^t 1 - \left(s^3 - \frac{3}{4}s^6 + \frac{3}{16}s^9 - \frac{1}{64}s^{12}\right) ds$$

$$= s - \frac{1}{4}s^4 + \frac{3}{28}s^7 - \frac{3}{160}s^{10} + \frac{1}{832}s^{13} \Big|_0^t$$

$$\boxed{\varphi_3 = t - \frac{1}{4}t^4 + \frac{3}{28}t^7 - \frac{3}{160}t^{10} + \frac{1}{832}t^{13}}$$

4. These are simple Calc I limits.

$$\text{If } x=1 \text{ then } \varphi_n(x)=1 \text{ and } \lim_{n \rightarrow \infty} 1 = 1$$

$$\text{If } x < 1 \text{ then } \varphi_n(x)=x^n \text{ and } \lim_{n \rightarrow \infty} x^n = 0.$$