

# Differential Equations: Project 4

Due: Monday, 1 July 2013

**Instructions:** Complete all problems in a neat and organized fashion on your own paper. If you use Wolfram|Alpha, a calculator, or any other resources, please state what you used it for. You will not lose any points for doing so, as long as you're honest about how and why you used it.

**1. and 2.** Use Euler's method to find approximate values of the solution of the initial value problems at  $t = 0.5, 1, 1.5, 2, 2.5, 3$ , with  $h = 0.1, 0.05, 0.025, 0.01$ . Make a table of the  $y$ -values, and plot all four (piecewise linear) curves on a single set of axes for each problem. You may use MatLab, Excel, or any other program that you like.

**1.**  $y' = 5 - \sqrt{y}; \quad y(0) = 2.$

**2.**  $y' = \frac{4 - ty}{1 + y^2}; \quad y(0) = -2.$

**3.** Let  $\varphi_0(t) = 0$  and use Picard's iteration method to calculate  $\varphi_1$ ,  $\varphi_2$ , and  $\varphi_3$ . Plot all three approximate solutions on a single set of axes (near the point  $(0, 0)$ ), and observe whether the iterates appear to be converging.

$$y' = 1 - y^3; \quad y(0) = 0$$

**4.** Let  $\varphi_n(x) = x^n$  for  $0 \leq x \leq 1$ . Show that

$$\lim_{n \rightarrow \infty} \varphi_n(x) = \begin{cases} 0, & 0 \leq x < 1, \\ 1, & x = 1. \end{cases}$$

This is an example of a sequence of continuous functions whose limit is not a continuous function.