Differential Equations: Project 2

Due: Monday, 17 June 2013

Instructions: Complete all problems in a neat and organized fashion on your own paper. If you use Wolfram|Alpha, a calculator, or any other resources, please state what you used it for. You will not lose any points for doing so, as long as you're honest about how and why you used it.

1. A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume of the raindrop as a function of time. Do *not* solve the DE.

2. Undetermined Coefficients. Here is an alternative way to solve the equation

$$\frac{dy}{dt} = ay - b. \tag{1}$$

(a) Solve the simpler equation

$$\frac{dy}{dt} = ay. \tag{2}$$

Call the solution $y_1(t)$.

(b) Observe that the only difference between these equations is the constant $-b$. Therefore it may seem reasonable to assume that the solutions of these two equations also differ only by a constant. Test this assumption by trying to find a constant k such that $y(t) = y_1(t) + k$ is a solution of equation (1).

(c) Solve equation (1) by integrating it directly, as we did in class. Compare your answer to the one in part (b).

Note: This method can also be used in some cases when the constant b is replaced by a function $g(t)$. It depends on whether or not you can guess the general form that the solution will take. We will study this method in detail in chapter 3.

3. Use the method of problem 2 to solve the equation

$$\frac{dy}{dt} = -ay + b.$$

4. Radium-226 has a half-life of 1620 years. Find the time period during which a given amount of this material is reduced by one-quarter.

5. Suppose that a building loses heat in accordance with Newton's law of cooling:

$$\frac{du}{dt} = -k(u - T),$$

where $k > 0$ is a constant, and T is the constant ambient temperature. Assume that $k = 0.15\,h^{-1}$, and the interior temperature is 70°F when a heating system fails. If the external temperature is 10°F, how long will it take for the interior temperature to fall to 32°F?

6. Verify that the functions $y_1(t) = e^{-3t}$ and $y_2(t) = e^t$ are solutions to the differential equation $y'' + 2y' - 3y = 0$.

7. Use the method of integrating factors to solve the differential equations.

(a) $y' - 2y = t^2 e^{2t}$

(b) $ty' + 2y = \sin t, \quad t > 0$

(c) $2y' + y = 3t^2$