Similarity

$$L(\underline{x}) = \begin{pmatrix} 2x_1 \\ x_1 + x_2 \end{pmatrix}$$

$$\left.\begin{array}{l} L(\underline{e}_1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ L(\underline{e}_2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array}\right\} \quad A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\{\underline{u}_1, \underline{u}_2\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$$

$$U = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad U^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

To determine the coords of $L(\underline{u}_1)$ wrt U,

$$U^{-1}(L(\underline{u}_1)) = U^{-1} A \, \underline{u}_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$U^{-1}(L(\underline{u}_2)) = U^{-1} A \, \underline{u}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\text{so} \quad B = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$$

A and B both represent the same trans.

$$B = U^{-1} A U$$

We say A and B are <u>similar</u> <u>matrices</u>.

<u>Defn</u>. Two matrices $A, B \in \mathbb{R}^{n \times n}$ are said to be <u>similar</u> if $\exists$ nonsingular S s.t. $B = S^{-1} A S$.

<u>Ex</u>. D on P_3 wrt $\{1, x, x^2\}$ and $\{1, 2x, 4x^2 - 2\}$

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \qquad S^{-1} = \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix} \qquad A = S^{-1} B S$$

Ex. $L: \mathbb{R}^3 \to \mathbb{R}^3 : \underline{x} \mapsto A\underline{x}$ for $A = \begin{pmatrix} 2 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix}$

$$\underline{y}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \underline{y}_2 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad \underline{y}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$L(\underline{y}_1) = \underline{0}$$
$$L(\underline{y}_2) = \underline{y}_2$$
$$L(\underline{y}_3) = 4\underline{y}_3$$

so $$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

also, $B = Y^{-1}AY$.

Sorry these notes are so sloppy. :(