

§1.1 Systems of Linear Equations

Defn. A linear eqn in n unknowns.

Defn. A system of m eqns in n unknowns

Defn. Solutions / Solution Set.

Examples of 2×2 - Use x_1, x_2 .

$$\begin{array}{lll} 3 \text{ cases: } & \text{i) } x_1 + x_2 = 2 & \text{ii) } x_1 + x_2 = 2 \\ & x_1 - x_2 = 2 & x_1 + x_2 = 1 \\ & & \text{iii) } x_1 + x_2 = 2 \\ & & -x_1 - x_2 = -2 \end{array}$$

Defn. Equivalent systems.

$$\begin{array}{ll} \text{Ex. } 3x_1 + 2x_2 - x_3 = -2 & 3x_1 + 2x_2 - x_3 = -2 \\ x_2 = 3 & -3x_1 - x_2 + x_3 = 5 \\ 2x_3 = 4 & 3x_1 + 2x_2 + x_3 = 2 \end{array}$$

We can collect the coefficients of these equations into an array to remove clutter. The array is called a matrix.

The system $\begin{cases} 3x_1 + 2x_2 - x_3 = -2 \\ -3x_1 - x_2 + x_3 = 5 \\ 3x_1 + 2x_2 + x_3 = 2 \end{cases}$ becomes the augmented matrix:

$$\left(\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & 1 & 5 \\ 3 & 2 & 1 & 2 \end{array} \right)$$

RE. Solve the system by performing all of the same operations we did to the system, to the matrix.

§1.2 - Row Echelon Form

Elementary Row Operations:

- I. Interchange two rows
- II. Multiply a row by a nonzero real number
- III. Replace a row by its sum w/ a multiple of another row.

These can be used to transform any matrix into row echelon form:

Defn (REF)

- I. The first nonzero entry in each nonzero row is 1.
- II. If row k does not consist entirely of zeros, the number of leading zeros in row $k+1$ is greater than the number of leading zeros in row k .
- III. If there are rows whose entries are all zeros, they lie below the rows having nonzero entries.

Examples of REF and non-REF

The act of using elementary row ops to transform a matrix into REF is known as Gaussian Elimination.

Examples.

$$\begin{aligned} 1. \quad & x_1 + x_2 = 1 \\ & x_1 - x_2 = 3 \\ & -x_1 + 2x_2 = -2 \end{aligned}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{array} \right)$$

No Sol'n

$$\begin{aligned} 2. \quad & x_1 + 2x_2 + x_3 = 1 \\ & 2x_1 + 4x_2 + 2x_3 = 3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

No sol'n

$$\begin{aligned} 3. \quad & -x_1 + x_2 - x_3 + 3x_4 = 0 \\ & 3x_1 + x_2 - x_3 - x_4 = 0 \\ & 2x_1 - x_2 - 2x_3 - x_4 = 0 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right) \text{ RREF!}$$

∞ -many sol'n

$$\begin{aligned} x_1 &= \alpha \\ x_2 &= -\alpha \\ x_3 &= \alpha \\ x_4 &= \alpha \end{aligned} \quad \{(\alpha, -\alpha, \alpha, \alpha) \mid \alpha \in \mathbb{R}\}$$