Name: Key

M511: Linear Algebra (Fall 2017)
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Chapter 2 Exam

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any notes or electronic devices. You may use a compass, straightedge, and colored pens/pencils.

Part I: True/False [4 points each]
Neatly write **T** *on the line if the statement is always true, and* **F** *otherwise [2 points]. In the space provided, give sufficient explanation of your answer [2 points].*

F **1.** $\det(A+B) = \det(A) + \det(B)$.

If A is any nonsingular matrix in $\mathbb{R}^{2\times 2}$, then put $B=-A$.

$\det(A+B) = 0$

$\det(A) + \det(-A) = 2\det(A) \neq 0$.

T **2.** $\det(A^T) = \det(A)$.

Expanding on row k of A^T is equivalent to expanding on column k of A.

F **3.** If $A \in \mathbb{R}^{n\times n}$ is a triangular matrix, then $\det(A) \neq 0$.

e.g., $A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$ is lower triangular but $\det(A) = 0$.

T **4.** If $A \in \mathbb{R}^{n\times n}$ is nonsingular, then $\operatorname{adj}(A)$ is nonsingular.

$A\left(\frac{1}{\det(A)}\operatorname{adj}(A)\right) = I$ if A is nonsingular implies

$\left(\frac{1}{\det(A)} A\right)\operatorname{adj}(A) = I$ whence $\operatorname{adj}(A)^{-1} = \frac{1}{\det(A)} A$.

F **5.** $(\mathbf{x}\times\mathbf{y}) = (\mathbf{y}\times\mathbf{x})$ for all $\mathbf{x},\mathbf{y} \in \mathbb{R}^3$.

$\bar{x} \times \bar{y} = -(\bar{y} \times \bar{x})$.

Part I: Written Problems [10 points each]
Follow all instructions exactly, and show enough work.

For questions **6 – 9** consider the matrix:

$$A = \begin{pmatrix} 2 & 3 & 5 \\ -2 & 4 & 2 \\ 1 & -1 & 3 \end{pmatrix}$$

6. Find and clearly label <u>all</u> minors of A.

$M_{11} = \begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}$ $M_{12} = \begin{pmatrix} -2 & 2 \\ 1 & 3 \end{pmatrix}$ $M_{13} = \begin{pmatrix} -2 & 4 \\ 1 & -1 \end{pmatrix}$

$M_{21} = \begin{pmatrix} 3 & 5 \\ -1 & 3 \end{pmatrix}$ $M_{22} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ $M_{23} = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$

$M_{31} = \begin{pmatrix} 3 & 5 \\ 4 & 2 \end{pmatrix}$ $M_{32} = \begin{pmatrix} 2 & 5 \\ -2 & 2 \end{pmatrix}$ $M_{33} = \begin{pmatrix} 2 & 3 \\ -2 & 4 \end{pmatrix}$

7. Find and clearly label <u>all</u> cofactors of A.

$A_{11} = 14$ $A_{12} = -(-8) = 8$ $A_{13} = -2$

$A_{21} = -14$ $A_{22} = 1$ $A_{23} = -(-5) = 5$

$A_{31} = -14$ $A_{32} = -14$ $A_{33} = 14$

Recall, for questions **6 – 9** consider the matrix:

$$A = \begin{pmatrix} 2 & 3 & 5 \\ -2 & 4 & 2 \\ 1 & -1 & 3 \end{pmatrix}$$

8. Compute $\det(A)$.

Expand on any row or column.

Row 1: $\det A = a_{11}A_{11} + a_{12}A_{13} + a_{13}A_{13}$

$= 2(14) + 3(8) + 5(-2)$

$= 28 + 24 - 10$

$= 42.$

9. Compute A^{-1}, provided it exists. If it does not exist, explain why.

$$A^{-1} = \frac{1}{\det(A)} C^T = \frac{1}{42}\begin{pmatrix} 14 & -14 & -14 \\ 8 & 1 & -14 \\ -2 & 5 & 14 \end{pmatrix}.$$

10. Use Cramer's Rule to solve the system of equations. Give your answer in simplified fraction form.

$$\begin{cases} 5x+7y &= 1 \\ -8x+3y &= -1 \end{cases}$$

$A = \begin{pmatrix} 5 & 7 \\ -8 & 3 \end{pmatrix}$ $\quad \bar{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\det(A) = 15 + 56 = 71$

$A_1 = \begin{pmatrix} 1 & 7 \\ -1 & 3 \end{pmatrix}$

$\det(A_1) = 3 + 7 = 10$

$A_2 = \begin{pmatrix} 5 & 1 \\ -8 & -1 \end{pmatrix}$

$\det(A_2) = -5 + 8 = 3$

$x = \dfrac{\det(A_1)}{\det A} = \dfrac{10}{71}$

$y = \dfrac{\det(A_2)}{\det A} = \dfrac{3}{71}$

Sol'n: $\boxed{(x,y) = \left(\frac{10}{71}, \frac{3}{71}\right)}$

11. Write det(A) as a simplified polynomial in x, where

$$A = \begin{pmatrix} x & 1 & 2 \\ -1 & x^2 & -3 \\ -2 & 3 & x^3 \end{pmatrix}$$

$\det(A) = x \begin{vmatrix} x^2 & -3 \\ 3 & x^3 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & x^3 \end{vmatrix} + 2 \begin{vmatrix} -1 & x^2 \\ -2 & 3 \end{vmatrix}$

$= x(x^5 + 9) - (-x^3 - 6) + 2(-3 + 2x^2)$

$= x^6 + 9x + x^3 + \cancel{6} \cancel{- 6} + 4x^2$

$= x^6 + x^3 + 4x^2 + 9x$

12. Find the values of λ that make A singular, where

$$A = \begin{pmatrix} 1-\lambda & \sqrt{3} \\ \sqrt{3} & -1-\lambda \end{pmatrix}.$$

$\det(A) = (1-\lambda)(-1-\lambda) - \sqrt{3}^2$

$= +(\lambda-1)(\lambda+1) - 3$

$= \lambda^2 - 1 - 3$

$= \lambda^2 - 4$

set $\det(A) = 0$,

$\lambda^2 - 4 = 0$

$\lambda^2 = 4$

$\boxed{\lambda = \pm 2}$

13. Find the determinant of A, where

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 \\ \sqrt{2} & 0 & 1 & 0 \\ -3 & -\frac{1}{2} & 56 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & -6 & 27 \\ 0 & 1 & -9 & \sqrt{3} \\ 0 & 0 & -2 & 115 \\ 0 & 0 & 0 & 5 \end{pmatrix}.$$

$A = LU$

$\det A = \det L \det U$

$= 1 \cdot (-20)$

$= -20.$