Name:________________________________

M511: Linear Algebra (Fall 2017)
Instructor: Justin Ryan
Chapter 5 Exam

Read and follow all instructions. You may not use any notes or electronic devices. Show enough work.

1. **[20 points]** Consider the subspace S of $\mathbb{R}^3$ spanned by $\mathbf{x} = (1, -1, 1)^T$. Find an orthonormal basis for $S^\perp$.

2. Let $(V, \langle\ ,\ \rangle)$ be an inner product space.

(a) [10 points] Prove the Cauchy-Schwarz-Bunyakovsky Inequality: $|\langle \mathbf{x}, \mathbf{y} \rangle| \leq \|\mathbf{x}\|\,\|\mathbf{y}\|$.

(b) [10 points] Use part **(a)** to prove the triangle inequality: $\|\mathbf{x}+\mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$.

3. **[20 points]** Consider the matrix

$$A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \\ 0 & 2 \end{pmatrix}.$$

Use the Gram-Schmidt process to find an orthonormal basis for $\mathrm{col}(A)$.

4. **[20 points]** Consider the subspace $S = \operatorname{span}\{1, x, x^2\}$ of $C[0,1]$ with inner product defined by

$$\langle f, g\rangle = \int_0^1 f(x)g(x)\,dx.$$

Find an orthonormal basis for S.

5. Consider the following vectors in $(\mathbb{R}^4, \cdot)$.

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} -2 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

(a) [10 points] State completely the Pythagorean Law for Inner Product Spaces.

(b) [10 points] Compute $\mathbf{p} = \mathrm{proj}_{\mathbf{y}}\mathbf{x}$, and show that $(\mathbf{x} - \mathbf{p}) \perp \mathbf{p}$.

(c) [10 points] Verify that the Pythagorean Law holds for $\mathbf{x}, \mathbf{p}$, and $\mathbf{x} - \mathbf{p}$.