Name:________________________________
M511: Linear Algebra (Fall 2017)
Instructor: Justin Ryan
Chapter 4 Exam

Read and follow all instructions. You may not use any notes or electronic devices.

Part I: True/False [4 points each]
Neatly write **T** *on the line if the statement is always true, and* **F** *otherwise [2 points]. In the space provided, give sufficient explanation of your answer [2 points].*

______ **1.** If $L : V \to V$ is a linear transformation and $\mathbf{x} \in \ker(L)$, then $L(\mathbf{v}+\mathbf{x}) = L(\mathbf{v})$ for all $\mathbf{v} \in V$.

______ **2.** Let $L : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. If $L(\mathbf{x}_1) = L(\mathbf{x}_2)$, then $\mathbf{x}_1$ must be equal to $\mathbf{x}_2$.

______ **3.** Let $L : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let A be the standard matrix representation of L. Then $\text{range}(L) = \text{row}(A)$.

______ **4.** Let $L : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let A be the standard matrix representation of L. Then $\ker(L) = \text{Null}(A)$.

______ **5.** The transformation of $\mathbb{R}^2$ that reflects each point in the plane over the line $y = 2x - 4$ is a linear transformation.

Part I: Written Problems [10 points each]
Follow all instructions exactly, and show enough work.

6–7. Consider the ordered bases $U = \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right\}$ and $V = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \end{pmatrix} \right\}$ of $\mathbb{R}^2$.

6. Consider the linear transformation defined by $L(\mathbf{u}_1) = 2\mathbf{u}_1 - \mathbf{u}_2$ and $L(\mathbf{u}_2) = \mathbf{u}_1 + 3\mathbf{u}_2$. Find the matrix representing L with respect to the basis U.

7. Find the matrix representing L with respect to the basis V.

8. Let $R_\ell : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects each vector over the line $\ell : y = x$. Find the matrix representing R_ℓ with respect to the standard basis.

9. Let $U = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$, and suppose $L(\mathbf{u}_1) = 2\mathbf{u}_1$ and $L(\mathbf{u}_2) = -4\mathbf{u}_2$. Find the matrix representing L with respect to the standard basis.

10–11. Suppose $L : \mathbb{R}^3 \to \mathbb{R}^2$ is given by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}.$$

10. Find a basis for $\ker(L)$.

11. Is L onto? Explain.

12–13. Consider the subspace $S = \operatorname{span}\left\{e^{-x}\cos x, e^{-x}\sin x\right\}$ of $C(\mathbb{R})$.

12. Find the matrix representing the derivative $D : f \mapsto f'$ on S.

13. Use the Fundamental Theorem of Calculus to compute

$$\int 3e^{-x}\cos x - 5e^{-x}\sin x \, dx$$

as a matrix product.