Name:______________________________
M511: Linear Algebra (Fall 2017)
Instructor: Justin Ryan
Chapter 3 Exam
Due by: ______________________________

Wichita State University

Read and follow all instructions. You may not use any notes or electronic devices. You may use a compass, straightedge, and colored pens/pencils.

Part I: True/False [4 points each]
Neatly write **T** *on the line if the statement is always true, and* **F** *otherwise [2 points]. In the space provided, give sufficient explanation of your answer [2 points].*

______**1.** Let $\mathbf{x} = (17, -4, 2)^T$, $\mathbf{v}_1 = (2, 1, -3)^T$, and $\mathbf{v}_2 = (1, -2, 4)^T$ in $\mathbb{R}^3$. Then $\mathbf{x}$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

______**2.** The vectors $\left\{(2,3,1)^T, (1,-1,2)^T, (7,3,8)^T\right\}$ span $\mathbb{R}^3$.

______**3.** Suppose V is a vector space and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ form a basis for V. Then $\{\mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3, \mathbf{v}_3 - \mathbf{v}_4, \mathbf{v}_4\}$ also form a basis for V.

______**4.** Any collection of four polynomials in P_4 form a spanning set for P_4.

______**5.** $S = \text{span}\left\{x, x^2, x|x|\right\}$ is a subspace of $C(\mathbb{R})$ of dimension 3.

Part I: Written Problems [10 points each]
Follow all instructions exactly, and show enough work.

6 – 9. For questions 6 through 9, consider the vector space P_4 with standard (ordered) basis $E = \{1, x, x^2, x^3\}$, and $V = \{1, (x-2), (x-2)^2, (x-2)^3\}$.

6. Show that V forms a basis of P_4.

7. Find the transition matrix from the ordered basis E to V.

8. Write the vector $p(x) = 8 - 12x + 6x^2 - x^3$ in V-coordinates.

9. Find an ordered basis $U = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ of P_4 for which

$$S = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

is the transition matrix from V to U. Give your answer as polynomials in P_4 (**not** in coordinates).

10. Consider the following vectors in $\mathbb{R}^2$,

$$\mathbf{u}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \text{and} \quad \mathbf{v}_2 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$$

Write $\mathbf{x} = 4\mathbf{u}_1 - 2\mathbf{u}_2$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

11. Consider the matrix,

$$A = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ 6 & -3 & -5 & -7 \end{pmatrix}$$

Find bases for the row space, column space, and null space of A. Clearly label each answer.

12. Consider a matrix $A \in \mathbb{R}^{3\times 5}$ whose columns $\mathbf{a}_1, \dots, \mathbf{a}_5$ satisfy $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_5$ are linearly independent, $\mathbf{a}_3 = \mathbf{a}_1 - \mathbf{a}_2$, and $\mathbf{a}_4 = 2\mathbf{a}_1 + \mathbf{a}_3$.

(*a*) What is the reduced row echelon form (RREF) of A?

(*b*) What is the column space of A?

13. Let $b \in \mathbb{R}$ be any real number, and define the operations $\oplus_b$ and $\otimes_b$ on $\mathbb{R}$ by:

$$\begin{aligned} x \oplus_b y &= x + y - b, \quad \text{and} \\ \alpha \otimes_b x &= \alpha x + (1-\alpha) b \end{aligned}$$

Is $(\mathbb{R}, \oplus, \otimes)$ a vector space? Give sufficient proof of your answer.