Name: M511: Linear Algebra (Fall 2017) Instructor: Justin Ryan Chapter 2 Exam



Read and follow all instructions. You may not use any notes or electronic devices. You may use a compass, straightedge, and colored pens/pencils.

Part I: True/False [4 points each]

Neatly write **T** on the line if the statement is always true, and **F** otherwise [2 points]. In the space provided, give sufficient explanation of your answer [2 points].

1. det(A + B) = det(A) + det(B).

2. $\det(A^T) = \det(A)$.

3. If $A \in \mathbb{R}^{n \times n}$ is a triangular matrix, then det $(A) \neq 0$.

4. If $A \in \mathbb{R}^{n \times n}$ is nonsingular, then $\operatorname{adj}(A)$ is nonsingular.

5. $(\mathbf{x} \times \mathbf{y}) = (\mathbf{y} \times \mathbf{x})$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$.

Part I: Written Problems [10 points each]

Follow all instructions exactly, and show enough work.

For questions **6** – **9** consider the matrix:

$$A = \begin{pmatrix} 2 & 3 & 5 \\ -2 & 4 & 2 \\ 1 & -1 & 3 \end{pmatrix}$$

6. Find and clearly label <u>all</u> minors of *A*.

7. Find and clearly label <u>all</u> cofactors of *A*.

Recall, for questions **6** – **9** consider the matrix:

$$A = \begin{pmatrix} 2 & 3 & 5 \\ -2 & 4 & 2 \\ 1 & -1 & 3 \end{pmatrix}$$

8. Compute det(*A*).

9. Compute A^{-1} , provided it exists. If it does not exist, explain why.

10. Use Cramer's Rule to solve the system of equations. Give your answer in simplified fraction form.

$$\begin{cases} 5x + 7y = 1\\ -8x + 3y = -1 \end{cases}$$

11. Write det(A) as a simplified polynomial in *x*, where

$$A = \begin{pmatrix} x & 1 & 2 \\ -1 & x^2 & -3 \\ -2 & 3 & x^3 \end{pmatrix}$$

12. Find the values of λ that make *A* singular, where

$$A = \begin{pmatrix} 1 - \lambda & \sqrt{3} \\ \sqrt{3} & -1 - \lambda \end{pmatrix}.$$

13. Find the determinant of *A*, where

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 \\ \sqrt{2} & 0 & 1 & 0 \\ -3 & -\frac{1}{2} & 56 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & -6 & 27 \\ 0 & 1 & -9 & \sqrt{3} \\ 0 & 0 & -2 & 115 \\ 0 & 0 & 0 & 5 \end{pmatrix}.$$