Name:________________________________

M511: Linear Algebra (Fall 2017)

Instructor: Justin Ryan

Chapter 1 Exam

Read and follow all instructions. You may not use any notes or electronic devices. You may use a compass, straightedge, and colored pens/pencils.

Part I: True/False [4 points each]

Neatly write **T** *on the line if the statement is always true, and* **F** *otherwise [2 points]. In the space provided, give sufficient explanation of your answer [2 points].*

______**1.** If A and B are $n \times n$ matrices, then $(A-B)^2 = A^2 - 2AB + B^2$.

______**2.** If $A = A^{-1}$, then either $A = I$ or $A = -I$.

______**3.** Every homogeneous linear system is consistent.

______**4.** If A and B are nonsingular $n \times n$ matrices, then $C = AB$ is also nonsingular.

______**5.** If A and B are nonsingular $n \times n$ matrices, then $C = (A+B)$ is also nonsingular.

Part I: Written Problems [10 points each]
Follow all instructions exactly, and show enough work.

6. Solve the linear system, if possible, using your favorite method.

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 - x_2 + x_3 = 3 \\ -x_1 + 2x_2 + 3x_3 = 7 \end{cases}$$

7. The augmented matrix is in reduced row echelon form. Find the solution of the corresponding system.

$$\left(\begin{array}{cccc|c} 1 & 5 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

8. Consider the matrices

$$A = \begin{pmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{pmatrix}.$$

Compute AB.

9. Given that $R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, show that $R^{-1} = R^T$.

10. The matrix $A = \begin{pmatrix} -2 & 4 \\ 6 & 8 \end{pmatrix}$ is nonsingular. Write A^{-1} as a product of elementary matrices.

11. Find the inverse of the given matrix. Do *not* use determinants.

$$A = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 3 & 0 \\ 2 & 5 & 0 \end{pmatrix}$$

12. Let $A \in \mathbb{R}^{n\times n}$ be a nonsingular matrix. Compute

$$\begin{pmatrix} A^{-1} \\ I \end{pmatrix} \begin{pmatrix} A & I \end{pmatrix}.$$

13. Find the *LU* Factorization of the matrix.

$$A = \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix}$$