Calculus III: Project 6

Due: Monday, 17 June 2013

Instructions: Complete all problems in a neat and organized fashion on your own paper. If you use Wolfram|Alpha, a calculator, or any other resources, please state what you used it for. You will not lose any points for doing so, as long as you're honest about how and why you used it.

1. Use Wolfram|Alpha to compute the iterated integrals:

$$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx \quad \text{and} \quad \int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy.$$

Do the answers contradict Fubini's theorem? What is happening?

2. (a.) In what way are Fubini's theorem and Clairaut's theorem similar? (b.) If $f(x,y)$ is continuous on $R = [a,b] \times [c,d]$ and

$$g(x,y) = \int_a^x \int_c^y f(s,t)\,dt\,ds$$

for $a < x < b$ and $c < y < d$, show that

$$g_{xy} = g_{yx} = f(x,y).$$

3. Use geometry or symmetry (what's the difference?) to evaluate the double integral:

$$\int\int_D \sqrt{R^2 - x^2 - y^2}\,dA$$

where D is the disc centered at the origin of radius R.

4. Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x xy\,dy\,dx + \int_1^{\sqrt{2}} \int_0^x xy\,dy\,dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy\,dy\,dx$$

into a single double integral; then evaluate the double integral.