

C3: Project 5: Solns

1. $F(x, y, z) = e^z - xyz$

$$\frac{\partial z}{\partial x} = \frac{-\partial F / \partial x}{\partial F / \partial z} = \frac{yz}{e^z - xy}$$

$$\frac{\partial z}{\partial y} = \frac{-\partial F / \partial y}{\partial F / \partial z} = \frac{xz}{e^z - xy}$$

2.
$$\begin{array}{c} f \\ \swarrow \quad \searrow \\ x \quad y \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ s \quad t \quad s \quad t \end{array}$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t} = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial s} \cdot \frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \right) \left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \right)$$

$$= \left(\frac{\partial f}{\partial x} \right)^2 - \left(\frac{\partial f}{\partial y} \right)^2$$

3. $F(x, y, z) = x^4 + y^4 + z^4 - 3x^2y^2z^2$

$$\nabla F = \langle 4x^3 - 6xy^2z^2, 4y^3 - 6x^2yz^2, 4z^3 - 6x^2y^2z \rangle$$

$$\nabla F(p) = \langle 4-6, 4-6, 4-6 \rangle = \langle -2, -2, -2 \rangle$$

$$T_p S: \nabla F(p) \cdot (\vec{n} - \vec{r}_0) = 0$$

$$\langle -2, -2, -2 \rangle \cdot \langle x-1, y-1, z-1 \rangle = 0$$

$$-2x+2 - 2y+2 - 2z+2 = 0$$

$$\boxed{x+y+z=3}$$

$$4. \quad f = xy, \quad \nabla f = \langle y, x \rangle, \quad \nabla f(3,2) = \langle 2, 3 \rangle$$

$$z - z_0 = \nabla f \cdot (\vec{r} - \vec{r}_0)$$

$$z = 6 + \langle 2, 3 \rangle \cdot \langle x-3, y-2 \rangle$$

$$z = 6 + 2x - 6 + 3y - 6$$

$$\boxed{z = 2x + 3y - 6}$$