C3: Project 4: Solns

1. a). $\partial_p R = \frac{g^2}{1+p^2g^4}$ $\partial_g R = \frac{2pg}{1+p^2g^4}$

b). $\partial_x u = y/z\, x^{y/z-1}$ $\partial_y u = \frac{1}{z} x^{y/z} \ln(x)$

$\partial_z u = -y/z^2\, x^{y/z} \ln(x)$

c.) $\partial_\alpha F = -\sqrt{\alpha^3+1}$ $\partial_\beta F = \sqrt{\beta^3+1}$

d.) $\partial_x f = \sin(y-z)$ $\partial_y f = x\cos(y-z)$ $\partial_z f = -x\cos(y-z)$

2. $\partial_m K = \frac{1}{2}v^2$ $\partial_v K = mv$ $\partial_v^2 K = m$

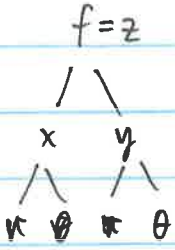
$$(\partial_m K)(\partial_v^2 K) = \left(\tfrac{1}{2}v^2\right)(m) = \tfrac{1}{2}mv^2 = K. \quad \blacksquare$$

3. $F(x,y,z) = xyz - \cos(x+y+z)$

$$\frac{\partial z}{\partial x} = \frac{-\partial F/\partial x}{\partial F/\partial z} = \frac{-(yz + \sin(x+y+z))}{xy + \sin(x+y+z)}$$

$$\frac{\partial z}{\partial y} = \frac{-\partial F/\partial y}{\partial F/\partial x} = \frac{-(xz + \sin(x+y+z))}{xy + \sin(x+y+z)}$$

4.



$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta$$

$$\frac{\partial^2 z}{\partial r^2} = \frac{\partial}{\partial x}\left[\frac{\partial z}{\partial x}\frac{\partial x}{\partial r}\right]\frac{\partial x}{\partial r} + \frac{\partial}{\partial x}\left[\frac{\partial z}{\partial y}\frac{\partial y}{\partial r}\right]\frac{\partial x}{\partial r}$$

$$+ \frac{\partial}{\partial y}\left[\frac{\partial z}{\partial x}\frac{\partial x}{\partial r}\right]\frac{\partial y}{\partial r} + \frac{\partial}{\partial y}\left[\frac{\partial z}{\partial y}\frac{\partial y}{\partial r}\right]\frac{\partial y}{\partial r}$$

$$= \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial r}\right)^2 + \frac{\partial^2 z}{\partial x \partial y}\frac{\partial y}{\partial r}\frac{\partial x}{\partial r} + \frac{\partial^2 z}{\partial y \partial x}\frac{\partial x}{\partial r}\frac{\partial y}{\partial r} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial r}\right)^2$$

$$= \frac{\partial^2 z}{\partial x^2}\cos^2\theta + 2\frac{\partial^2 z}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 z}{\partial y^2}\sin^2\theta$$

After a similar computational mess, you should get the result. This is beyond tedious and I did it on the board.

5. $u(t,x) = f(x+at) + g(x-at)$

Show: $u_{tt} = a^2 u_{xx}$

$u_t = a f'(x+at) - a g'(x-at)$

$u_{tt} = a^2 f''(x+at) + a^2 g''(x-at)$

$u_x = f'(x+at) + g'(x-at)$

$u_{xx} = f''(x+at) + g''(x-at)$

$a^2 u_{xx} = a^2 f''(x+at) + a^2 g''(x-at)$ ✓