Calculus III: Project 4

Due: Monday, 1 July 2013

Instructions: Complete all problems in a neat and organized fashion on your own paper. If you use Wolfram|Alpha, a calculator, or any other resources, please state what you used it for. You will not lose any points for doing so, as long as you're honest about how and why you used it.

1. Find all first partial derivatives of the functions

a. $R(p, q) = \arctan(pq^2)$

b. $u = x^{y/z}$

c. $F(\alpha, \beta) = \int_\alpha^\beta \sqrt{t^3 + 1}\, dt$

d. $f(x, y, z) = x\, \sin(y - z)$

2. The kinetic energy of a body with mass m and velocity v is $K = \frac{1}{2}mv^2$. Show that

$$\frac{\partial K}{\partial m}\frac{\partial^2 K}{\partial v^2} = K.$$

3. Use the implicit differentiation formulas given in class to find $\partial z/\partial x$ and $\partial z/\partial y$ for

$$xyz = \cos(x + y + z).$$

4. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

5. Show that any function of the form $u(t, x) = f(x + at) + g(x - at)$ is a solution of the *wave equation*

$$u_{tt} = a^2\, u_{xx}.$$