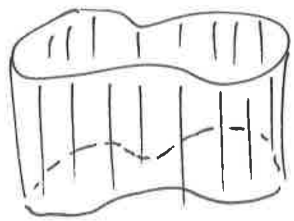
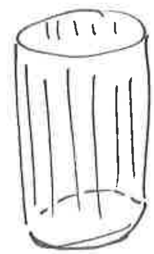
10.6 Quadratic Surfaces & Cylinders

To sketch more complicated surfaces than lines and planes it will be necessary to look at the traces (or cross-sections) of the surfaces.

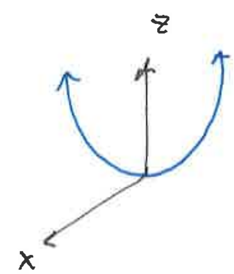
A cylinder is a surface that consists of all straight, parallel lines passing thru a plane curve:



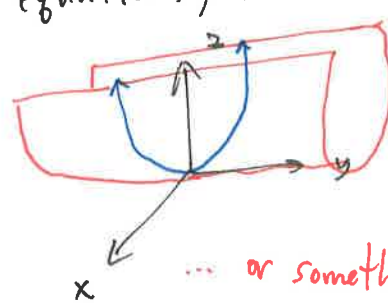


Ex. $z = x^2$

In the x-z plane, this is a parabola.



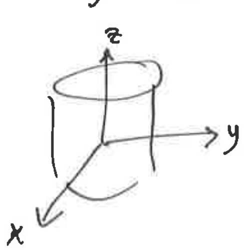
Since y can be any real number (it's not in equation), then this forms a parabolic cylinder.

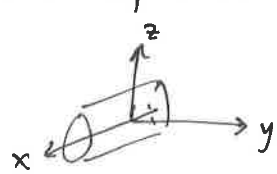


... or something.

Ex. The "usual" cylinders look something like:

$x^2 + y^2 = 1$, $y^2 + z^2 = 1$, etc.





* talk about traces here!

A quadratic surface is the graph of a second order equation in $\mathbb{R}^3$.

The most general such equation is:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

Phew...

However, by translations and rotations (ie., rigid transformations) this can be written in one of two forms:

$$\begin{cases} Ax^2 + By^2 + Cz^2 + J = 0 \\ Ax^2 + By^2 + Iz = 0 \end{cases}$$

Quadratic surfaces in $\mathbb{R}^3$ are the counterpart to conic sections in $\mathbb{R}^2$ (see §9.5 or my College algebra notes).

For these examples we will need to use traces.

Ex. $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

We take a trace by setting one variable equal to a constant and looking at the 2-dimensional ~~contour~~ plane formed by the other two variables. This is similar to the slicing method of finding volumes of solids of revolution.

set $z = k$, then

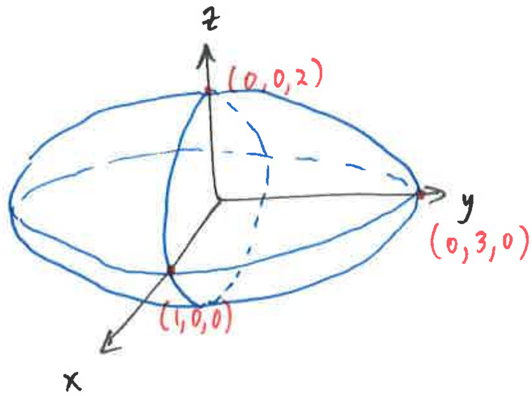
$$x^2 + \frac{y^2}{9} = 1 - \frac{k^2}{4}$$

← this is an ellipse in the resulting xy-plane for $-2 < k < 2$, and is undefined outside that interval. "Largest" when $k = 0$.

Similarly, putting $x=k$ yields

$$\frac{y^2}{9}+\frac{z^2}{4}=1-k^2 \quad : \quad \text{ellipse for } -1<k<1, \text{ and}$$

$y=k$, $\quad x^2+\frac{z^2}{4}=1-\frac{k^2}{9} \quad : \quad$ ellipse for $-3<k<3$.



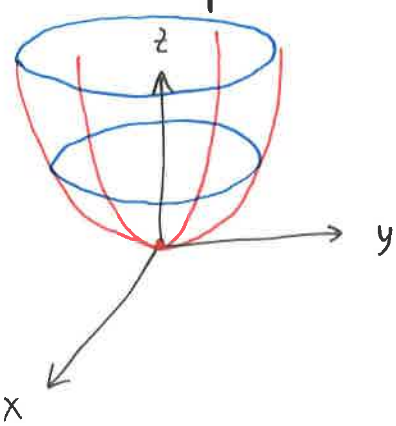
This surface is called an ellipsoid.

Ex. $z=4x^2+y^2$

$z=k \qquad k=4x^2+y^2 \qquad$ is an ellipse for $k>0$.

$x=k \qquad z=4k^2+y^2 \qquad$ is a parabola in the yz-plane for all k.

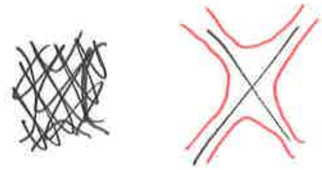
$y=k \qquad z=4x^2+k^2 \qquad$ is a parabola in the xz-plane for all k



This is an elliptic paraboloid.

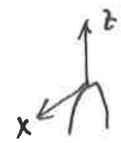
__Ex.__ $z = y^2 - x^2$

$z = k$: hyperbola in the xy-plane

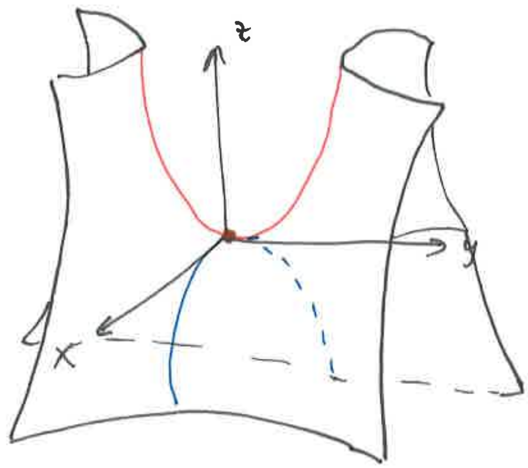


$x = k$: $z = y^2 - k^2$ parabola in yz-plane opening up

$y = k$: $z = k^2 - x^2$ parabola in xz plane opening down



All together now...

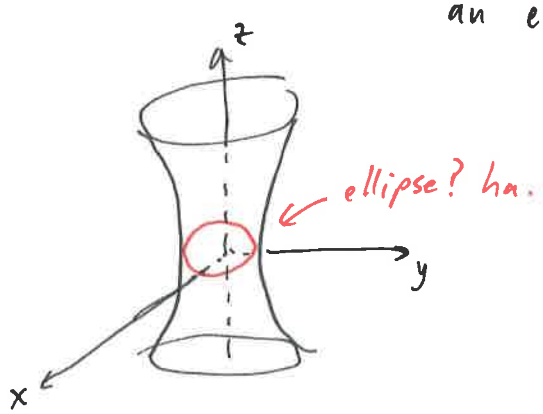


... or something.

looks like a saddle. In fact, the origin is called a saddle point.

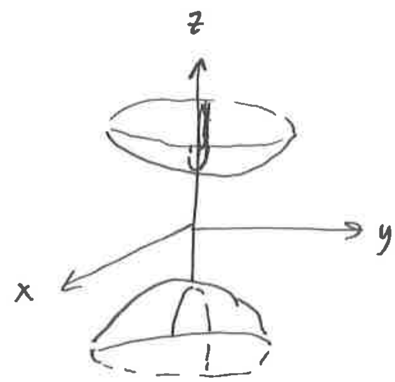
This is called a hyperbolic paraboloid.

__Ex.__ $\frac{x^2}{4} + y^2 - \frac{z^2}{4} = 1$. FTIS traces. Two ~~hyperabolas~~ hyperbolas and an ellipse makes: hyperboloid of one sheet



<u>Ex</u>. FTIS: $\frac{-x^2}{9} - \frac{y^2}{4} + \frac{z^2}{4} = 1$

<u>Hyperboloid of two sheets</u>



they're getting worse...

* Look at and memorize(?) the table on pg. 557.

The only one we didn't draw is the cone. See if you can find the traces and make sense of it.

<u>Ex</u>. Classify the surface:

$$x^2 + 2z^2 - 6x - y + 10 = 0$$

$$x^2 - 6x \quad - y + 2z^2 + 10 = 0$$

$$(x-3)^2 - 9 - y + 2z^2 + 10 = 0$$

$$(x-3)^2 - y + 2z^2 = 1$$

$(x-3)$ is just a translation, so this is an

<u>elliptic paraboloid</u>.