

Calculus III: Project 1

Due date: Thurs, 31 Jan 13

Instructions: On your own paper, solve the following problems in a clean, neat, clear, organized, legible, *etc* manner. Your assignment will be graded on the presentation and validity of your work.

Assume that all vectors are in \mathbb{R}^3 . Prove the following properties of the dot and cross products:

1. $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$. Give a geometric explanation for why this is true.

2. $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.

3. $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} \\ \vec{a} \cdot \vec{d} & \vec{b} \cdot \vec{d} \end{vmatrix} = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$.

4. Suppose that all sides of a quadrilateral are equal in length and opposite sides are parallel. Use vector methods to show that the diagonals are perpendicular. [Hint: Draw a picture or two.]