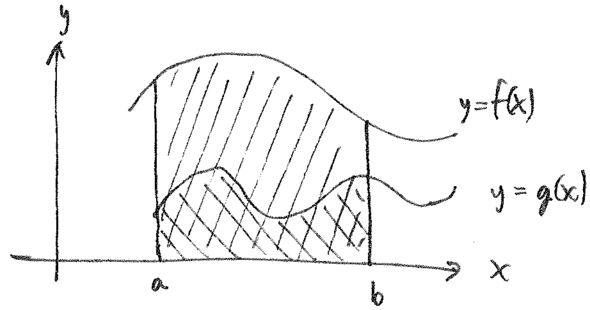
Chapter 7: Applications of Integration

7.1: Area between curves



Area between f and g = area under f − area under g.

or $\quad$ Area $= \int_a^b [f(x) - g(x)]\,dx$

Ex. Area between $y=e^x$ and $y=x$ from $x=0$ to $x=1$.

$y=e^x$ is on top.

$$\text{Area} = \int_0^1 e^x - x\,dx = e^x - \tfrac{1}{2}x^2 \Big|_0^1 = e - \tfrac{1}{2} - 1 + 0$$
$$= \boxed{e - 3/2}$$

Ex. Find the area enclosed by the parabolas $y=x^2$, $y=2x-x^2$

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x=0 \quad x=1$

$y=x^2$ is not on top

$$A = \int_0^1 2x^2 - 2x\,dx = \tfrac{2}{3}x^3 - x^2 \Big|_0^1$$
$$= \left|\tfrac{2}{3} - 1\right| = \left|-\tfrac{1}{3}\right| = \tfrac{1}{3}$$

Ex. $y=x-1$, $y^2 = 2x+6$ Find area enclosed.

$x = y+1 \qquad x = \frac{1}{2}y^2 - 3$

$\frac{1}{2}y^2 - 3 = y+1$

$\frac{1}{2}y^2 - y - 4 = 0$

$y^2 - 2y - 8 = 0$

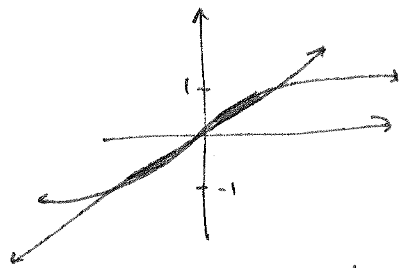
$(y-4)(y+2)$

$y=4 \quad y=-2$

$$\int_{-2}^{4} (y+1) - \left(\tfrac{1}{2}y^2 - 3\right) dy$$
$$= \int_{-2}^{4} -\tfrac{1}{2}y^2 + y + 4\,dy$$
$$= -\tfrac{1}{6}y^3 + \tfrac{1}{2}y^2 + 4y \Big|_{-2}^{4} = \dots = \boxed{18}$$

Ex. $x=y$, $x=y^3$ find area enclosed

$$y=y^3 \quad\Big\}\ y=\pm 1, 0$$
$$y^3-y=0$$
$$(y+1)(y-1)y=0$$



These pieces have the same area.

So, $A = 2\int_0^1 y-y^3\,dy$

$$=2\left(\tfrac{1}{2}y^2-\tfrac{1}{4}y^4\right)\Big|_0^1$$

$$=2\left(\tfrac{1}{2}-\tfrac{1}{4}\right) = 2\left(\tfrac{1}{4}\right)=\boxed{\tfrac{1}{2}}$$

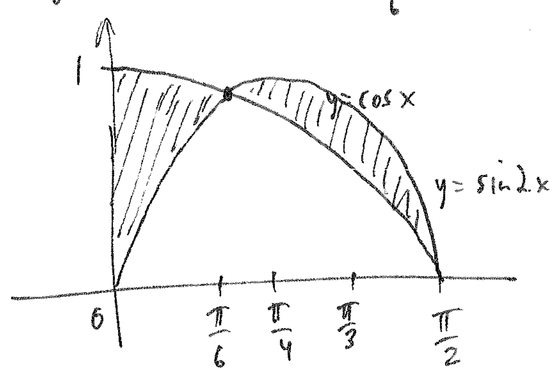
$y=\sin 2x \qquad y=\cos x \qquad 0 \to \frac{\pi}{2}$

$$\sin 2x = \cos x$$
$$2\sin x\cos x = \cos x$$
$$2\sin x\cos x - \cos x = 0$$
$$\cos x\,(2\sin x - 1) = 0$$

$\cos x = 0 \qquad \sin x = 1/2$

$\Rightarrow x=\frac{\pi}{2} \qquad \Rightarrow x=\frac{\pi}{6}$

$$\int_0^{\pi/6} \cos x-\sin 2x\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x-\cos x\,dx$$



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