Math 243: Calculus II
Exam 2: Ch 6

Monday, 17 September 2012

Name: KEY

Instructions: Complete all problems, showing all work. Problems are graded not only on whether the answer is correct, but also if the work leading up to the answer is correct. Simplify as necessary. Leave any answers involving π or irreducible square roots in terms of such (no rounded off decimals).

1. – 7. Evaluate the definite and indefinite integrals. If the integral is divergent, write "divergent".

1. $\displaystyle\int \frac{5}{x^2+6x+34}\,dx$ $= 5\displaystyle\int \frac{1}{(x+3)^2+5^2}\,dx = \frac{5}{5}\arctan\left(\frac{x+3}{5}\right)+C$

$x^2+6x+34 = x^2+6x+9 \quad +25$ $\qquad = \boxed{\arctan\left(\frac{x+3}{5}\right)+C}$

$= (x+3)^2+5^2$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \sec^2\theta\,d\theta$ $= \displaystyle\lim_{t\to\frac{\pi}{2}^-}\int_0^t \sec^2\theta\,d\theta$

$= \displaystyle\lim_{t\to\frac{\pi}{2}^-} \tan\theta\Big|_0^t$

$= \displaystyle\lim_{t\to\frac{\pi}{2}^-} \tan t - \tan 0$

$= +\infty - 0 = +\infty$

$\Rightarrow \boxed{\text{divergent}}$

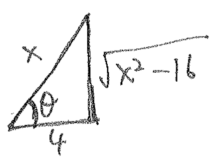
3. $\int x^2 \sin x\, dx \;=\; \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

	u	dv
$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$
$-$	0	$\cos x$

4. $\int \frac{1}{x^2\sqrt{x^2-16}}\, dx$

let $x = 4\sec\theta \;\Rightarrow\; \sec\theta = \frac{x}{4}$

$dx = 4\sec\theta\tan\theta\, d\theta$



$\Rightarrow \sin\theta = \frac{O}{H} = \frac{\sqrt{x^2-16}}{x}$

$$\Rightarrow \int \frac{4\sec\theta\tan\theta\, d\theta}{16\sec^2\theta\sqrt{16(\sec^2\theta-1)}} = \frac{1}{16}\int \frac{\cancel{4}\cancel{\sec\theta}\,\cancel{\tan\theta}\, d\theta}{\sec^{\cancel{2}}\theta\;\cancel{4}\;\cancel{\tan\theta}} = \frac{1}{16}\int \cos\theta\, d\theta$$

$$= \frac{1}{16}\sin\theta + C = \boxed{\frac{\sqrt{x^2-16}}{16x} + C}$$

5. $\int_1^e \ln x \, dx$. Show all work.

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$\int_1^e \ln x \, dx = x \ln x \Big|_1^e - \int_1^e dx$

$= (x \ln x - x)\Big|_1^e$

$= e\cancel{\ln e} - \cancel{e} - 1\cancel{\ln(1)} + 1$

$= \boxed{1}$

6. $\int \frac{4}{x^2 - 9}\, dx \quad = \frac{-2}{3}\int \frac{1}{x+3}\,dx + \frac{2}{3}\int \frac{1}{x-3}\,dx$

$\frac{4}{x^2 - 9} = \frac{A}{x+3} + \frac{B}{x-3}$

$4 = A(x-3) + B(x+3)$

$x = 3: \quad 4 = 6B \quad B = \frac{2}{3}$

$x = -3 \quad 4 = -6A \quad A = \frac{-2}{3}$

$= \frac{-2}{3}\ln|x+3| + \frac{2}{3}\ln|x-3| + C$

$= \boxed{\frac{2}{3}\ln\left|\frac{x-3}{x+3}\right| + C}$

7. $\int \cos^2\theta \, d\theta = \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$

$$= \boxed{\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C}$$

8. Use the comparison theorem to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{1+e^{-x}}{x}\, dx$$

$\frac{1+e^{-x}}{x} > \frac{1}{x}$ for $x \geq 1$

$\int_1^\infty \frac{1}{x}\, dx$ diverges by p-test

so, by comp. test: $\boxed{\int_1^\infty \frac{1+e^{-x}}{x}\, dx \text{ also diverges}}$