Calculus II Project:
Time Dilation and Lorentz Contraction

Due date: Wed, 5 Dec 12

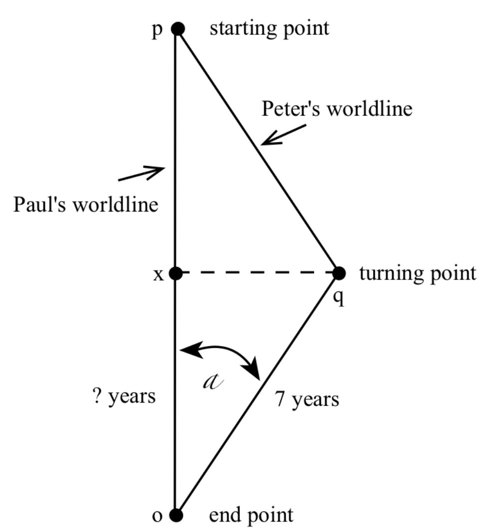
In Special Relativity there are remarkable phenomena called *time dilation* and *Lorentz contraction*. Time dilation states that time slows down as an observer moves at high speeds (close to the speed of light). Lorentz (or length) contraction states that objects moving close to the speed of light appear shorter to a fixed observer.

Problems:

1. The Twin Paradox. On their 21^{st} birthday, Peter leaves his twin brother Paul behind on a freely falling spaceship and departs at a constant relative speed of $v = 24/25$ (in geometric units), for a free fall of 7 years of his proper time (that is, time as he experiences it). Then he turns around and returns symmetrically in another 7 years. When he returns, he is 35 years old. How old is Paul?

Using the figure below, Paul's age is equal to $21 + 2(ox)$, where ox is the length (in years) of the line segment connecting the points o and x. You'll need the following hyperbolic trig identities:

$$\cosh(\alpha) = \frac{ox}{op}, \quad \text{and} \quad \cosh(\alpha) = \frac{1}{\sqrt{1-v^2}}.$$



2. An Einstein Train. Let L_0 be the length of an object at rest. Lorentz contraction says that the length of this object, when moving at a velocity v, is given by $L = L_0\sqrt{1 - v^2}$.

Suppose a train of rest length 200 m travels along a straight stretch of track past a station of rest length 100 m. Thus when the train is parked at the station, it is twice as long. Suppose that, on a particular trip, the train travels at a speed of $\sqrt{3}/2$ past the station without stopping. Calculate the length of the train as seen by the stationmaster at the station, and the length of the station as seen by the conductor of the train. Compare the lengths of the train and station from each person's perspective. [Recall that the station is at rest to the stationmaster, but not to the conductor. Likewise, the train is at rest to the conductor, but not to the stationmaster.]

3. Traveling the Universe. Suppose that you get in a freely falling spaceship and fly around the universe at a constant relative speed for 40 years. You might think, since the speed of light is the "speed limit of the universe," that you would be able to travel at most 40 light years in this time. However, time dilation and length contraction makes it possible for you to travel further.

Use the two previous problems to calculate exactly how many light years you will travel if you keep a constant speed of $v = 3/5$ during your journey. What if you travel at a speed of $v = 399/401$?

Note. Length contraction was actually hypothesized by Hendrik Lorentz *before* Einstein developed his theory of special relativity; hence the name Lorentz contraction. So the idea that time and space are intimately intertwined was around before Einstein, it just took his brilliance to put it all together into one elegant theory.

Reference. O'Neill, Barrett, *Semi-Riemannian Geometry,* Academic Press: New York, 1983.