

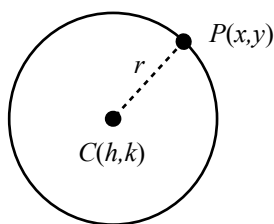
Calculus II Project: Conic Sections

Due date: Wed, 5 Dec 12

In this project you will use the “coordinate invariant” definitions of a circle and parabola to derive their Cartesian equations.

Definition. A *circle* is the set of all points equidistant from a single fixed point.

Problem 1. Referring to the figure below, a circle is the set of all points P that satisfy the equation $d(C, P) = r$, where d is the usual distance function in \mathbb{R}^2 . Use this definition to derive the standard Cartesian equation of a circle with radius r , centered at (h, k) .



Definition. A *parabola* is the set of all points equidistant from a fixed point F (the *focus*) and a fixed line ℓ (the *directrix*).

Problem 2.a. Draw a picture of a parabola together with its focus and directrix. Label the vertex, focus, directrix, any point P on the curve, and the point Q on the directrix that is the projection of P . Also draw and label the *latus rectum*. What is the length of the latus rectum?

2.b. Suppose the vertex of the parabola is the origin $V(0, 0)$. Then its Cartesian equation should be $y = ax^2$. Derive this formula from the picture, and determine exactly the constant a . What is the geometric meaning of a ?

2.c. Now let the vertex be $V(h, k)$, and repeat the above calculation to derive the general Cartesian equation of a parabola.