Project 7.4 : Arc Length Competition.

Problem: Find a function y=f(x) with the properties that
i. f(0) = 0
ii. f(1) = 0
iii. f(x) >= 0 for all 0<x<1
such that the area under y=f(x) between 0 and 1 is 1, and the arc length from 0 to 1 is as small as possible.

Solution:
Start with a quadratic function f(x) = a(x-x1)(x-x2)
The zeros of f should be 0 and 1, so f(x) = ax(x-1)
Since f should be positive between 0 and 1, then a must be negative. For now, choose a = -1.
So our first attempt at finding a function is f(x) = x - x^2.
The area is given by

In[5]:=
```
Integrate[x-x^2,{x,0,1}]
Integrate[x - x^2, {x, 0, 1}]
```

Out[5]= $\frac{1}{6}$

Since we need the area to be 1, we need to multiply our function by 6. Now f(x) = 6(x-x^2) satisfies all of the criteria. Let's find it's arc length.

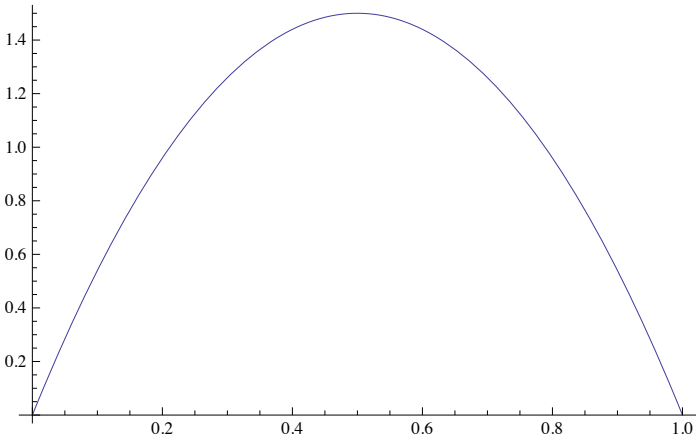
In[14]:=
```
arc length of y = 6*(x-x^2) from 0 to 1
Result
N[NIntegrate[Sqrt[1 + 36 * (1 - 2 * x) ^ 2], {x, 0, 1}]]
```

Out[14]= 3.24903

So the length of this curve is L(f) = 3.24903. The curve looks like this:

In[12]:=
```
Plot[y = 6 * (x - x^2), {x, 0, 1}]
```

Out[12]=



Let's try the other function from class. Let g(x) = sin(pi*x). The area is given by

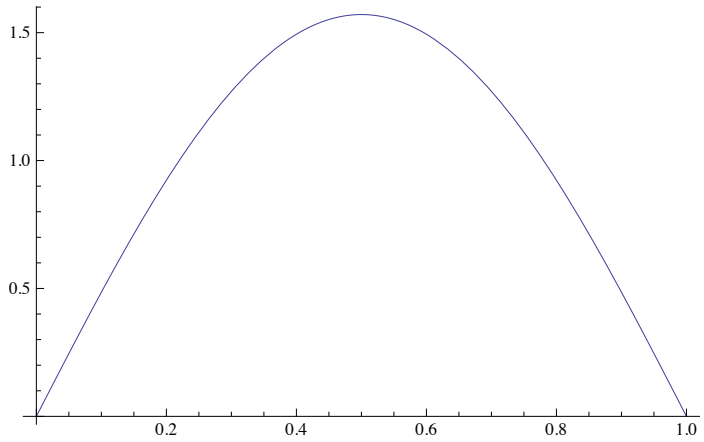
In[7]:=
```
Integrate[sin(Pi∗x),{x,0,1}]
Integrate[Sin[Pi * x], {x, 0, 1}]
```

Out[7]= $\frac{2}{\pi}$

So, to make g satisfy the area requirement we must set g(x) = (pi/2)*sin(pi*x). The cure looks like this:

In[17]:=
```
Plot[y = (Pi / 2) * Sin[Pi * x], {x, 0, 1}]
```

Out[17]=



The arc length is given by:

In[8]:=
```
arc length of y = (Pi/2)∗Sin(Pi∗x) from 0 to 1
Result
N[NIntegrate[Sqrt[1 + (Pi^4 * Cos[Pi * x]^2) / 4], {x, 0, 1}]]
```

Out[8]= 3.3655

In conclusion, L(f) = 3.24903 < L(g) = 3.3655.

So my entry into the Arc Length competition is f(x) = 6*(x - x^2) with an arc length of 3.24903. Can you beat me? :-)