§1.8 - Continuity

Recall the nice properties of polynomials, rational functions, trig functions.

Algebraic idea of continuity.

Calculus defn of continuity.

- at a point
- on an interval

"A function is continuous if and only if it commutes with limits."

Theorems about continuous functions: $\pm, \cdot, \div$, scalar mult, $\circ$

Very Important Theorem:

Intermediate Value Theorem. Suppose f is continuous on the closed interval $[a,b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$.
Then there exists a real number c in (a,b) such that $f(c) = N$.

Ex. $f(x) = 4x^3 - 6x^2 + 3x - 2$
Show that f has a real zero between 1 and 2.

Ex. Is there a number that is exactly one more than its cube?

Ex. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$ Remove the discontinuity at $x = 2$.

* Ex. Prove that f is continuous at a if and only if

$$\lim_{h \to 0} f(a+h) = f(a).$$