

§1.6 - Limit Laws

* - We will not ~~put~~^{put} everything from this section on the chalkboards. Please read the theorems in blue boxes.

Theorem. Let k be a constant and

$$\lim_{x \rightarrow a} f(x) = L$$

$$\lim_{x \rightarrow a} g(x) = M$$

Then,

$$1. \lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$$

$$3. \lim_{x \rightarrow a} [k f(x)] = k \cdot \lim_{x \rightarrow a} f(x) = kL$$

$$4. \lim_{x \rightarrow a} [f(x) g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) = L \cdot M$$

$$5. \lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M} \quad \text{as long as } M \neq 0.$$

$$6. \lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n = L^n$$

$$7. \lim_{x \rightarrow a} k = k$$

$$8. \lim_{x \rightarrow a} x = a$$

$$9. \lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{L},$$

provided $L > 0$ if n is even.

Examples. Polynomial
Rational

"Factorable" Rational

Some important theorems:

Thm 1.5.2 - If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist, then

$$\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$$

Thm 1.6.3 - Squeeze Theorem

If $f(x) \leq g(x) \leq h(x)$ when x is near a , except possibly at a , and

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L,$$

Then $\lim_{x \rightarrow a} g(x) = L$.

Ex. $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$

— Use Geogebra.