Name: Key

M242: Calculus I (Fall 2017)
Instructor: Justin Ryan
Chapter 1 Exam

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

Part I: True/False [2 points each]

Neatly write ***T*** *if the statement is always true, and* ***F*** *otherwise.*

F **1.** $\lim_{x\to 4}\left(\frac{2x}{x-4}-\frac{8}{x-4}\right)=\lim_{x\to 4}\frac{2x}{x-4}-\lim_{x\to 4}\frac{8}{x-4}$

F **2.** Let f be a function satisfying $f(a)=k$. Then $\lim_{x\to a} f(x)=k$.

T **3.** If p is a polynomial, then $\lim_{x\to b} p(x)=p(b)$.

T **4.** The equation $x^4-6x^2+5=0$ has a root in the interval $(0,2)$.

F **5.** If $|f|$ is continuous at a, so is f.

Part II: Multiple Choice [5 points each]

Select the best answer and write its corresponding letter neatly on the given line.

B **6.** Compute $\lim_{x\to 0}\cos(x+\sin x)$

A. 0 **B.** 1

C. $\cos(1)$ **D.** $\frac{\pi}{2}$

C **7.** Compute $\lim_{x\to 5}\frac{x^2-25}{x-5}$

A. 0 **B.** 5

C. 10 **D.** Does Not Exist

A **8.** Compute $\lim_{x\to 0} x^2\cos\left(\frac{2\pi}{x}\right)$

A. 0 **B.** 1

C. $+\infty$ **D.** Does Not Exist

B **9.** Compute $\lim_{\theta\to \frac{\pi}{2}^+} \tan\theta$

A. $+\infty$ **B.** $-\infty$

C. 0 **D.** Does Not Exist

C **10.** Compute $\lim_{x\to 3} \frac{x^2+2x-5}{x+2}$

A. 0 **B.** -2

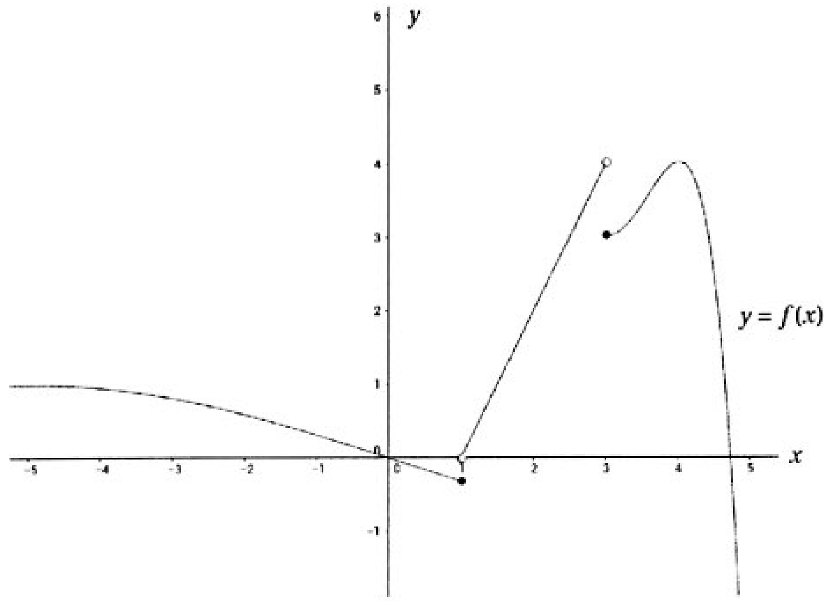
C. 2 **D.** Does Not Exist

A **11.** Compute $\lim_{v\to 4^+} \frac{4-v}{|4-v|}$

A. -1 **B.** 1

C. 0 **D.** Does Not Exist

12–13. Use the graph of the function f to compute the limits.



C **12.** $\lim_{x \to 3^-} f(x)$

A. 0 **B.** 3

C. 4 **D.** Does Not Exist

D **13.** $\lim_{x \to 1} f(x)$

A. 0 **B.** $-\frac{1}{4}$

C. 3 **D.** Does Not Exist

Part III: Written Problems [10 points each]

Complete all problems, showing enough work.

14. Does the function $f(x) = \cos x - x^3$ have a real zero between 0 and $\frac{\pi}{2}$? Explain.

f is continuous on $\mathbb{R}$.

$f(0) = \cos(0) - 0^3 = 1 - 0 = 1 > 0$

$f(\frac{\pi}{2}) = \cos(\pi/2) - (\frac{\pi}{2})^3 = 0 - \frac{\pi^3}{8} < 0$

Thus the IVT applies, and f must have a real zero between 0 and $\frac{\pi}{2}$.

15. Compute $\lim\limits_{\theta \to 0} \dfrac{\sin^2(2\theta)}{\theta^2}$. Show enough work.

$$\lim_{\theta \to 0} \frac{\sin^2(2\theta)}{\theta^2} = \lim_{\theta \to 0} \left(\frac{2 \sin 2\theta}{2\theta}\right)^2 = 2^2 \lim_{\theta \to 0} \left(\frac{\sin 2\theta}{2\theta}\right)^2 =$$

$$= 4 \cdot \left(\lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta}\right)^2 = 4 \cdot \left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2 = 4 \cdot (1)^2 = 4.$$

let $x = 2\theta$

so $\lim\limits_{\theta \to 0} x = 0$

16. You wish to prove that $\lim\limits_{x \to 2} 14 - 5x = 4$. If you fix $\varepsilon > 0$, what should you set δ equal to in order to finish the proof? Show enough work.

Fix $\varepsilon > 0$ and suppose $0 < |x-2| < \delta$.

Then,

$|14 - 5x - 4| = |-5x + 10| = |-5(x-2)| = |-5||x-2| = 5|x-2| < 5\delta$

So put $\varepsilon = 5\delta$. Then $\delta = \frac{\varepsilon}{5}$.

17. Compute $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$. Show enough work.

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h} =$$

$$= \lim_{h\to 0} \frac{h\left(3x^2+3xh+h^2\right)}{h}$$

$$= \lim_{h\to 0} \left(3x^2+3xh+h^2\right)$$

$$= 3x^2 + 3x\cdot 0 + 0^2$$

$$= 3x^2.$$

18. Let $F(x) = \begin{cases} x^2 - 2 & x < 0 \\ k & x = 0. \\ -2\cos(x) & x > 0 \end{cases}$

What must k equal in order for F to be continuous at 0? Explain.

For F to be continuous at $x=0$, then $F(0)$ must equal $\lim_{x\to 0} F(x)$.

Compute,

$$\left.\begin{aligned} &\lim_{x\to 0^-} F(x) = \lim_{x\to 0^-} x^2-2 = (0^-)^2-2 = -2 \\ \text{and}\quad &\lim_{x\to 0^+} F(x) = \lim_{x\to 0^+} -2\cos(x) = -2\cos 0 = -2 \end{aligned}\right\}$$

So define $k=-2$.