Name: Key
M242: Calculus I (Fall 2017)
Instructor: Justin Ryan
Chapter 2 Exam

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain!

Part I: True/False [2 points each]
*Neatly write **T** if the statement is always true, and **F** otherwise.*

T **1.** If f is differentiable at $x = a$, then f is continuous at $x = a$.

F **2.** If f is continuous at $x = a$, then f is differentiable at $x = a$. ex. $f(x) = |x|$

F **3.** Suppose f' exists. The domain of f' coincides with the domain of f. ex. $f(x) = \sqrt{x}$ or $f(x) = |x|$

T **4.** Polynomial functions are differentiable on $(-\infty, \infty)$.

F **5.** Rational functions are differentiable on $(-\infty, \infty)$. ex. $f(x) = \frac{1}{x}$

Part II: Multiple Choice [5 points each]
Compute the derivatives of the given functions. Select the best answer and write its corresponding letter neatly on the given line.

B **6.** $f(x) = ax^2 + bx + c$ where a, b, c are real numbers

A. $2x + 1$ **B.** $2ax + b$

C. $2a + b$ **D.** $x^2 + 2ax + b + x$

A **7.** $y = (x^2 + x^3)^4$

A. $4(x^2 + x^3)^3(2x + 3x^2)$ **B.** $4(x^2 + x^3)^3$

C. $4(2x + 3x^2)^3$ **D.** $(2x + 3x^2)^4$

I marked everyone's answer correct. My mistake.

D 8. $g(x) = x^2 \sin(\pi x)$

A. $2x + \pi\cos(\pi x)$ **B.** $2x\cos(\pi x)$

C. $x^2\cos(\pi x)$ **D.** $2x\sin(\pi x) + \pi x^2\cos(\pi x)$

C 9. $f(t) = \dfrac{t^4 - 1}{t^4 + 1}$

A. $\dfrac{t^4 - 1}{(t^4+1)^2}$ **B.** $\dfrac{4t^3}{4t^3}$

C. $\dfrac{8t^3}{(t^4+1)^2}$ **D.** 1

A 10. $y = \tan(\sin\theta)$

A. $\sec^2(\sin\theta)\cos\theta$ **B.** $\sec^2(\cos\theta)$

C. $\sec^2\theta + \cos\theta$ **D.** $\sec^2(\sin\theta)$

D 11. $f(x) = \cot^3(x)$

A. $-3\cot^3(x)\csc(x)$

B. $3\cot^2(x)$

C. $3\cot^2(x)\csc(x)$

D. $-3\cot^2(x)\csc^2(x)$

A 12. $f(x) = x^2\sqrt{x^2+1}$

A. $\dfrac{3x^3+2x}{\sqrt{x^2+1}}$

B. $\dfrac{2x^2}{\sqrt{x^2+1}}$

C. $2x + \dfrac{x}{\sqrt{x^2+1}}$

D. $\dfrac{2x}{2\sqrt{2x}}$

D 13. $y = \dfrac{3x^4 - x^3 + 4x^2}{x^5}$

A. $\dfrac{12x^3 - 3x^2 + 8x}{5x^4}$

B. $\dfrac{3x^4 - x^3 + 4x^2}{x^{10}}$

C. $\dfrac{12x^3 - 3x^2 + 8x}{x^{10}}$

D. $\dfrac{-3x^2 + 2x - 12}{x^4}$

Part III: Written Problems [10 points each]
Complete all problems, showing enough work.

14. Use the limit definition of derivative to compute $f'(x)$. You must use the limit definition to receive credit.

$$f(x) = \frac{1}{x}$$

$$f'(a) = \lim_{x\to a} \frac{\frac{1}{x} - \frac{1}{a}}{x-a} = \lim_{x\to a} \frac{\frac{a-x}{ax}}{x-a} = \lim_{x\to a} \frac{-\cancel{(x-a)}}{ax\cancel{(x-a)}} = \lim_{x\to a} \frac{-1}{ax} = \frac{-1}{a^2}.$$

So $f'(x) = \frac{-1}{x^2}$.

15. Find the Taylor polynomial T_3 of $f(x) = \sin(x)$ at $x = 0$.

$f(x) = \sin x$ $\quad f(0) = 0$

$f'(x) = \cos x$ $\xrightarrow{x=0}$ $f'(0) = 1$

$f''(x) = -\sin x$ $\quad f''(0) = 0$

$f'''(x) = -\cos x$ $\quad f'''(0) = -1$

$$T_3(x) = \underbrace{\cancel{f(a)}}_{0} + f'(a)(x-a) + \underbrace{\cancel{\frac{f''(a)}{2}}}_{0}(x-a)^2 + \frac{f'''(a)}{6}(x-a)^3$$

$$\boxed{T_3(x) = x - \frac{1}{6}x^3}$$

16. Use a linear approximation or differentials to approximate the value of $\sqrt{9.1}$.

$f(x) = \sqrt{x}$ $\quad a=9$ $\quad f(9)=3$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad f'(9) = \frac{1}{6}$

$Lf(x) = 3 + \frac{1}{6}(x-9)$

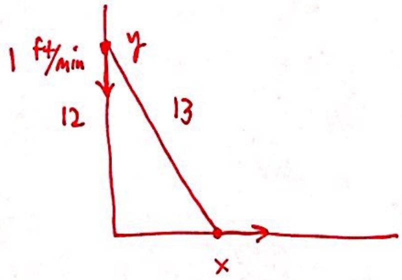
$Lf(9.1) = 3 + \frac{1}{6}(9.1-9)$

$= 3 + \frac{1}{60}$

$= \frac{181}{60}$

So, $\boxed{\sqrt{9.1} \approx \frac{181}{60}}$

17. A 13 ft ladder is sliding down a wall at a rate of $1\frac{\text{ft}}{\text{min}}$. Find the rate that the base of the ladder is sliding away from the wall when the top of the ladder is 12 ft from the ground. Include the proper units in your final answer.



$$x^2+y^2=13$$

$$\frac{d}{dt}\left[x^2+y^2=13\right]$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-1\ \frac{\text{ft}}{\text{min}}$$

$$\frac{dx}{dt}=\frac{-y}{x}\frac{dy}{dt}$$

when $y=12$, $x=5$, so

$$\boxed{\frac{dx}{dt}=\frac{-12}{5}(-1)=+\frac{12}{5}\ \frac{\text{ft}}{\text{min}}.}$$

18. Find the slope-intercept equation of the normal line to the circle $x^2+y^2=1$ at the point $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$.

$$x^2+y^2=1$$

$$\frac{d}{dx}\left[x^2+y^2=1\right]$$

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

So $\frac{dy}{dx}=\frac{-\sqrt{2}/2}{-\sqrt{2}/2}=1.$

Therefore $\frac{1}{dy/dx}=1$ also.

<u>Normal Line</u>:

$$y=y_1+\left(-1/\tfrac{dy}{dx}\right)(x-x_1)$$

$$y=-\frac{\sqrt{2}}{2}-1\left(x-\frac{\sqrt{2}}{2}\right)$$

so, $\boxed{y=-x}$